Robotic Grasping and Contact: A Review

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Abstract

In this paper, we survey the field of robotic grasping and the work that has been done in this area over the last two decades, with a slight bias toward the development of the theoretical framework and analytical results in this area.

1 Introduction

The human hand is used in a variety of ways. In particular, the three most important functions are to explore, to restrain objects, and to manipulate objects (relative to the wrist and to the palm). The first function falls within the realm of *haptics*, an active research area in its own merits [25]. We will not attempt an exhaustive coverage of this area. The work in robot grasping has tried to understand and to emulate the other two functions. We will distinguish between the task of restraining objects, sometimes called *fixturing*, and the task of manipulating objects with fingers (in contrast to manipulation with the robot arm), sometimes called *dexterous manipulation.*

While grippers and fixtures have been used extensively in industry, one can argue that the field of robot grasping started with the work of Asada and Hanafusa [1] and Salisbury's first attempts to develop a three-fingered robotic hand [35]. The most sophisticated multifingered hand built to date is the Utah-MIT hand [22]. While it was a beautifully designed and versatile hand with 32 actuators, it also illustrated some of the difficulties in robot control and the complexity of the problems underlying grasping and contact. In contrast to this work, there have been a number of efforts that have instead focussed on reduced-complexity, special-purpose multifingered hands. One of the first such attempts was a three fingered hand powered by four actuators [68] that was designed to grasp by *enveloping.*

Enveloping grasps [66], in contrast to fingertip grasps, are formed by wrapping the fingers (and the palm) around the fingers. Enveloping grasps are superior in terms of restraining objects. In fact, this is easily seen in human grasping where fingertips and distal phalanges are used in fingertip grasps for fine manipulation, while the inner parts of the hand (palm and proximal phalanges) are used in enveloping grasps for restraint [8, 21]). Variations of this basic theme are also seen in grippers designed for the so called *whole arm grasps* [56] and *power grasps* [37]. It is also interesting to note that in spite of the wide range of grippers and robotic hands used in industry, they are almost exclusively used for restraint and for fixturing, and not for dexterous manipulation.

2 Closure properties of grasps

Consider an object grasped at N contacts. It is generally assumed in the literature that all contacts are point contacts and idealizations such as a line or surface contact can be approximated by two or more point contacts. Each point contact can be modeled as either a frictionless point contact, a frictional point contact, or a soft contact [55]. A frictionless contact is defined as a contact in which the finger (or effector/fixture) can only exert a force along the common normal at the point of contact. A frictional contact (sometimes referred to as a point contact with friction) is defined as a contact that can transmit both a normal force and a tangential force, while a soft contact also allows the finger to exert a pure torsional moment about the common normal at the point of contact.

At each contact, the three forces and three moments can be modeled by a 6×1 vector, called the wrench vector. Let the unit wrenches corresponding to the normal force, the tangential force, and the moment about the normal be denoted by ${}^i w_N$, ${}^i w_T$, and ${}^i w_\theta$, respectively. The corresponding magnitudes or intensities are given by ${}^i c_N$, ${}^i c_T$, and ${}^i c_\theta$. We can construct a vector of wrench intensities, c, which can be partitioned into c_N, c_T, and c_θ in an obvious fashion. Finally, let W be the wrench matrix consisting of all unit wrenches, and g be the (possibly zero) known external wrench. Now we are in a position to establish some basic definitions and properties of grasps that are useful for analysis.

The analysis of mechanical fixtures and jigs goes back to the work of Reuleaux [52] in 1875. The first important concept is that of equilibrium. A grasped object is in equilibrium if and only if [55]: (a) For all i, the contact forces and moments satisfy the contact constraints; and (b) The object is in static equilibrium:

$$Wc + g = 0$$

where the contact constraints include the inequality:

$${}^i c_N \geq 0,$$

and the inequalities that characterize the frictional forces and moments. Typically, Coulomb's law with an appropriately chosen coefficient of friction is used to limit the tangential forces:

$$\left|{}^{i}c_T\right| \leq {}^{i}\mu_T \; {}^{i}c_N.$$

Similarly, a Coulomb like frictional law can be applied to a frictional moment [35]:

$$\left|{}^{i}c_\theta\right| \leq {}^{i}\mu_\theta \; {}^{i}c_N.$$

Alternatively a coupled linear model of the friction cones [4] for tangential force and the torsional moment, which has been shown in some cases to be more accurate, can be used.

A grasp is force closed, if and only if it is in equilibrium for any arbitrary wrench $\hat{w}$ [44]. Thus, force closure implies, for any arbitrary wrench $\hat{w}$, there exists an intensity vector λ satisfying the contact constraint inequalities, such that

$$W\lambda = \hat{w}.$$

Note that the intensity λ can be different from the actual intensity c, just as the hypothesized $\hat{w}$ can be different from g. The reader is advised to refer to [65] for other interpretations of force closure.

Form closure is defined as a condition of complete restraint in which the grasped body can resist any external disturbance wrench, irrespective of the magnitude of the contact forces [29, 44]. Form closure is a stronger condition than force closure. More formally, a grasp is defined as form closed if and only if it is force closed with frictionless contacts. Equivalently, a frictionless grasp with N unilateral wrenches is defined as form closed if and only if [65], there exist $\lambda > 0$, such that

$$W\lambda = 0,$$

and W is full rank.

Salisbury [55] derived a simple analytical procedure to test for either form or force closure. Trinkle [65] provided a quantitative test for form closure based on linear programming that provides a measure of how far the grasp is from losing the property of form closure.

Reuleaux proved in 1875 that planar bodies require at least four frictionless contacts to achieve form closure. On the other hand, at least seven frictional contacts are required for form closure in the spatial case [54, 29]. Reuleaux also showed that there exist various geometrical shapes in which it is impossible to completely constrain by any number of frictionless surface contacts. Selig and Rooney [59] classified Reuleaux's surfaces based on group theory. They derived a simple classification of surfaces which cannot be grasped. Mishra, Schwartz, and Sharir [38] were the first to set an upper bound of twelve frictionless contacts needed to achieve form closure on any spatial object with a piecewise smooth boundary (except Reuleaux's surfaces), and six frictionless contacts for any piecewise smooth planar object. Markenscoff, Ni, and Papadimitriou [34] showed that any planar object with a piecewise smooth boundary (except a circle) can be always form closed with no more than four frictionless contacts. For spatial objects, they showed that in most cases (including all polyhedra) a spatial object can be form closed with only seven frictionless contacts.

Constructive procedures for placing contacts on given objects to achieve form–closure have attracted much attention in the literature [34, 44, 49, 50]. Since this is also very relevant to fixturing, the reader is referred to a survey paper on fixture that appears in these proceedings [16].

Because the analysis in the literature discussed thus far is based on a first order kinematic model, the closure properties of a grasp depend only the locations of the contact points and the contact normals, but not on the shape of the object and the contacting effectors. If second order effects are examined, it is necessary to incorporate a model of the curvature of the contacting surfaces. Even if a grasp is not form (force) closed, second order effects may guarantee the closure of the grasp. The formulation of such second order effects is discussed in [18, 53, 67]). Higher order kinematic effects require the derivatives of the curvature and Christoffel symbols characterizing the contacting surfaces [57]. While third and higher order closure properties have not been formally defined, we now have a roadmap of how this line of work might proceed. We can now claim that such closure related properties are well understood and well-known techniques for analysing grasps exist.

3 Force Analysis

A crucial problem in robot grasping is the choice of grasp forces so as to avoid, or minimize the risk of, slippage. The *internal forces* [55], also sometimes called the *interaction forces* or the *squeeze forces*, are the contact forces lying in the nullspace of the grasp matrix W. It turns out, there is a unique decomposition of the grasp forces into the *equilibrating forces*, or the forces that lie in the range space of W, and the internal forces [28, 42]. The problem of choosing contact forces, or actuator forces if the kinematics and/or dynamics of the fingers are considered, so as to realize the required manipulating forces required by the task, while imposing constraints to prevent slip, is often referred to as the *force distribution* problem. This problem also occurs in other robot systems with closed kinematic chain, including legged locomotion systems and cooperating manipulators [46, 26, 31, 43, 69, 42].

The problem of determining the appropriate internal forces can be posed as an optimization problem. Different approaches including linear programming [26], pseudo inverse [28], and mathematical programming [26] have been proposed. Depending on the formulation of the problem, properties such as convexity [3] can be exploited to yield efficient solutions. Similarly if the nonlinear friction constraints can be written as positive-definiteness constraints [6], the inequalities can be written in terms of a standard linear matrix inequality (LMI) problem, for which efficient off-the-shelf software exists.

It is important to note that force closure does *not* guarantee stability. Any definition of stability must regard the grasp as a dynamic system and describe the properties of the dynamic system when it is perturbed from an equilibrium configuration. The easiest test for stability is based on a quasi-static model [60]. Consider a grasp with elastic fingers [1, 8]. We can derive all contact forces from a potential function $\phi(q)$, where q describes the configuration of the dynamic system. If for every small perturbation δq from an equilibrium grasp, $\delta\phi > 0$, by Lagrange's theorem the equilibrium configuration is said to be stable. If, on the other hand, a small perturbation δq such that $\delta\phi < 0$ exists, the equilibrium grasp is unstable [19]. Thus the stability of the grasp is effected by the local properties of the geometry of the grasp and the force distribution, in addition to the locations of the contact points and the contact normals.

Salisbury [55] established a basic framework for testing the stability of a grasp. He showed that a grasp is stable if the stiffness matrix (which characterizes the grasp) is positive definite. Cutkosky and Wright [10] looked at the specific case of a two-fingered grasp, and established relationships between local geometry and stability with a simple decoupled model for the stiffness of the servo control loops. This work was subsequently extended to more general grasps with simple models of compliance of fingers [8, 9]. A similar line of work shows that by modeling each finger-object contact as a virtual spring, force closed grasps can always be made stable by adjusting the applied forces at each finger [44].

The curvature of the object and the effector at the contact point has a significant effect on grasp stability [44, 41, 18]. In non force closed grasps, the contact grasp stability, or the tendency of the grasped object to return to the same point of contact, is determined by the the relative curvature, in addition to the position and the normals at the contact points [41]. There are three other groups that have pursued second order models of grasp mobility and stability. Trinkle, Farahat, and Stiller [65, 67] Trinkle and his coworkers developed a general formulation for the stability of non-force closed grasps for polygonal objects [65, 67]. They developed the concepts of first and second order stability cells, neighborhoods of force closed grasps which are also form closed and not form closed but stable, respectively. Howard and Kumar [19] incorporated the second order effect of contact curvature, the compliance at each contact, the magnitude of contact forces and friction, in their analysis of stability. They also provided a systematic classification of stable, but not force closed grasps. Rimon and Burdick [53] developed the concept of first and second order mobility of a grasp. A first order immobile grasp is form closed, while a second order immobile grasp is one that is not form closed but is immobile when the curvature of the contacting surfaces is considered.

A more exhaustive analysis of stability must include the control laws used for actuating the hand joints, the mechanical impedance of the system, and the contact models that describe the interactions between the fingers and the objects. While the work in [18] is a starting point, a detailed analysis has never been carried out.

4 Contact models

Kinematics of contact Contact kinematics is a study of the relationship between the location of the point of contact as a function of the relative motion of two contacting bodies. The first fundamental work in this area is due to Cai and Roth [7], who studied rigid planar bodies in point contact. They derived a relationship for the rates of change of the location of the point of contact as a function of the angular and linear velocities and accelerations of the contacting bodies. Montana [40] provided a more formal description of the configuration space associated with two contacting bodies, and derived the equations of kinematic contact that relate the time derivatives of contact coordinates with the relative angular and linear velocities. These equations include terms that depend on the curvature of the contacting bodies. Sarkar, Kumar, and Yun [57], extended this work to include acceleration terms. By using intrinsic geometric properties for the contacting surfaces, they showed the explicit dependence on the Christoffel symbols and their time derivatives. This set of results is directly relevant to dexterous manipulation [45], to the analysis of higher order closure properties [53], and to stability analysis [19].

Contact compliance The importance of modeling the finger-object contact and the role of compliance in grasping has been stressed by many researchers [1, 8, 60]. However, it is particularly difficult to model the relationship between small object/finger displacements and changes in contact forces arising from these displacements.

Such contact problems have been studied extensively in the solid mechanics community in the context of rail-wheel interaction [24] and analysis of ball and roller bearings [23]. There are difficulties even in establishing the uniqueness and existence of solutions of elastic bodies in static contact [12], and tractable analytical models are, in general, very difficult to come by. Hertz's model [23]can be used to predict the pressure distribution across each contact patch when the contacts are frictionless and non-conformal. Hertzian contact theory is probably the most widely used analytical contact model, and variations of this are used in [19, 53].

Because friction is central to robotic grasp, the Hertzian contact model has proved to be inadequate in many cases. Sinha and Abel [60] proposed an elastic contact stress model for finger-object contacts in multifingered grasping and a variational approach for quasistatic analysis. Wang, Kumar, and Abel [70] proposed a similar approach for dynamic analysis. They developed a mathematical programming approach for frictional, elastic contacts as well as viscoelastic contacts in which the inertial forces due to the deformations at the contacts are neglected. While such distributed parameter models yield accurate results, the solutions require computation-intensive numerical methods. A possible simplification is provided by the Winkler elastic foundation model [23], and the lumped parameter visco-elastic models used in [17, 27, 62] provide the simplest model for simulation and analysis.

One of the very hard problems is getting an accurate and tractable model of contact compliance, particularly in the tangential direction. This is recognized to be a difficult problem in the mechanics literature as well [23]. In addition to this, a tractable and accurate model of friction, one that accurately predicts slip and one that lends itself to stability analysis, is currently not available. Both these fundamental problem areas are crucial to robotic grasping and contact analysis.

5 Measures of grasp performance

Recent work in the literature has tried to develop quality measures for grasps. One such measure can be derived from the conditioning of the grasp or wrench matrix W and is directly connected with the closure properties of the grasp [31]. In a similar fashion, other structural properties can be derived from the characteristic matrices, for example, controllability and observability [51].

When an object is restrained or grasped with multiple effectors, there are two, often conflicting, measures of grasp performance. First, if the fixtures can be accurately positioned, the system's ability to reject wrench disturbances is a measure of grasp stability. The grasp stiffness matrix, or a frame invariant measure of the minimum grasp stiffness [5], provides one choice for a performance metric. This assumption of being able to accurately position the end-effector is extensively used in the fixturing and grasping literature. However, when there are errors in positioning and orienting the end-effectors, it is important to choose a grasp so that the system performance is insensitive to these positioning errors. Thus, it also makes sense to minimize the dependence of grasp forces on such positioning errors.

Howard and Kumar [19] develop the theory needed to combine the stiffness matrices at each contact to calculate a grasp stiffness matrix. While the signs of the eigenvalues allow a test of grasp stability, the eigenvalues themselves are not invariant with respect to changes in reference frames [18]. Bruyninckx *et al.* [5] develops a frame invariant measure of stability that is based on the grasp stiffness matrix and a metric on the Euclidean group. Lin develops a frame-invariant quality measure that essentially minimizes the "object deflection" when the grasped object is subject to force disturbances [32]. The basic idea here is to scale the eigenvalues measuring the rotational stiffness by a characteristic distance to an edge of the object. Thus it is possible to develop a scaled stiffness matrix and the smallest eigenvalue of the scaled matrix characterizes the system.

The focus in the above work is to quantify the ability of a fixture to reject disturbances due to external forces on the workpiece [11]. This is clearly a measure of performance that is relevant. However, the robustness of a grasp to errors in positioning the effectors has not been addressed in this literature. Sugar and Kumar develop a second measure of performance that characterizes this robustness and discuss an approach to optimizing fixtures based on both measures [63]. In this connection, the control of grasping and the effects of of uncertainties are particularly important.

Unfortunately most of these measures are based on the assumptions of small perturbations: displacements, forces and errors. There is no question that more global measures would be more useful. For example, in stability analysis, a figure relating to the size of the basin of attraction of the equilibrium, indicating how large a perturbation can be without causing instability would be desirable. However, the nonsmooth nature of grasp dynamics (because of the unilateral constraints on displacements and forces) has made a thorough analysis very difficult.

6 Dynamics of the hand

It is interesting that much of the literature in grasping actually ignores the kinematics of the fingers or the articulations that are involved in contacting the object. While Reuleaux's problem of form closure justifiably focuses on the geometry of the object and the arrangement of contacts, it is difficult to analyze a grasp without modeling the dynamics, or at least the kinematics, of the fingers and the interaction of the fingers with the object.

Trinkle *et. al.* explore the kinematics of enveloping grasps [66] using the restrictive but conservative assumption of frictionless contacts. The kinematics of fingers with two or three point contacts with fingertips and palms have been studied by [48, 13]. While the analysis of form–closure is intrinsically geometric, force–closure is tightly linked to the kinematics and characteristics of the end–effector. In fact, it is possible that a geometric analysis of a grasp may predict force–closure, but a careful analysis of the kinematics may reveal that this is not the case [18]. Definitions of force–closure that take into account the kinematics of the gripping device were proposed in [3], along with an exact algorithm for testing such property. Yoshikawa proposes a new set of definitions for closure properties, including what he calls active and passive closures, to explicitly model the properties of the grasping mechanism [73]. Unfortunately, much of this, and other related work [20] is based on instantaneous kinematics.

Modeling of the fingers is particularly important when end–effectors that have fewer degrees–of–freedom than necessary to impart arbitrary motions/forces at all contacts. Such *kinematically defective* grasps are common in simple industrial grippers. The hand Jacobian matrix is not full rank, it is not possible to command an arbitrary set of grasp forces [2]. This is usually the case in all power grasps. The modeling of the kinematics and manipulability of whole–hand manipulation in such systems is discussed in [51]. Intuitively, the more a grasp is defective, the more robust it is in restraining an object with respect to external disturbances, but the lower the "manipulability", and also lower the sensitivity to positioning errors. However, a case-by-case analysis is necessary for optimal power grasps [3].

Many open problems remain to be solved in order to be able to design robot hands to effectively exploit defectivity to increase grasp robustness and reduce hardware complexity. Among these, perhaps the most important is the need for a reliable estimate of contact compliance, arising with statically indeterminate grasps. This will then allow the calculation of contact forces, and the development of models that relate joint displacements and torques to contact forces.

7 Dynamics

The ability to predict the dynamic behavior of a grasp with a given model including the control algorithms, is critical to the design of the grasp. In multifingered grippers, as in legged locomotion systems, multi-arm systems, and other constrained robot systems, several limbs are used to constrain and manipulate an object [28, 31, 37]. The dynamic analysis and the simulation (the prediction of motion given the external forces and moments on the system) of such systems is central to the design of such systems and the development of control algorithms [71, 58].

When there are contacts between nominally rigid bodies, the constraints that arise in such situations are called unilateral constraints because the contact forces (and relative displacements) can be defined so that they are non-negative. Featherstone [14], Lotstedt [33] and Mason and Wang [36] pointed out some of the inconsistencies which arise when rigid body models are used with Coulomb's empirical law of friction in unilateral systems. For example, if we consider the simulation of a rod sliding along a rough ground in a plane with a single contact, there are configurations in which no solutions (that are consistent with the constraints) exist, and others in which the solution is not unique. Wang, Kumar, and Abel [70, 71] performed a dynamic analysis of the peg-in-the-hole insertion problem and showed that there was a range of parameters during two-point-contact for which there were either no solutions or two solutions for the accelerations. Quasi-static analysis is also known to exhibit such inconsistencies [19].

The inconsistencies and ambiguities in the dynamic analysis of frictional contacts have been attributed to the approximate nature of Coulomb's model and to the incorrect assumption of rigidity. Recently, there has been some attention in the robotics community on overcoming these shortcomings by using rigid body models to predict the gross motion while using compliant contact models to predict the contact forces and the local deformations [27].

One of the main difficulties that is present in multi-fingered grasps, and a feature that is particularly true of such grasps as power grasps and enveloping grasps, is that the number of independent contact forces is much larger than the number of actuators. Thus, from a controllability standpoint, not all the contact forces are controllable.

The analysis of statically indeterminate grasps or grasps in which there is no unique solution to the inital value problem is simply not possible unless one explicitly models the compliance at the contacts [8, 19, 44, 27]. Of course such contact models tend to be more complex and the parameters are more difficult to identify. Further, it is harder to simulate systems in which the time scale for the dynamics of contact interactions is significantly different from the time scale of rigid body dynamics [47, 62]. Thus, although efficient, approximate algorithms for "impulsive dynamic simulation" that incorporate approximate impact models for collisions are available [39], it is very difficult to write accurate simulators for dexterous and fine manipulation where the contact forces may be finite and the results may be sensitive to the parameters in the contact model.

8 Concluding Remarks

This paper presented a survey of work in robotic grasping over the last twenty years. It is impossible to do justice to all the work in this area, particularly because of the breadth of the field and its close connection to dexterous manipulation, fixturing, and haptics. We chose to focus on issues that are central to the mechanics of grasping and the finger–object contact interactions. In addition, the review mainly addressed research that has established the theoretical framework for grasp analysis, simulation and synthesis. Because of the limitations on space, we have not given the algorithmic aspects, and the applications the attention that they deserve. We hope that this paper complements the other survey papers that appear in this conference [16, 45].

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