Temperature of a human body immersed into extremely cold ocean water

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Abstract

The present essay is about an example of energy flux. It is considered the case of a naked human body fully immersed into cold water, such as it might be deep in the ocean. Starting from the general heat equation it was found the decreasing function of the body temperature and it was observed depending on which physical quantities it decreases more quickly or more slowly. It is considered that the body is losing heat both by conduction and convection.

Introduction

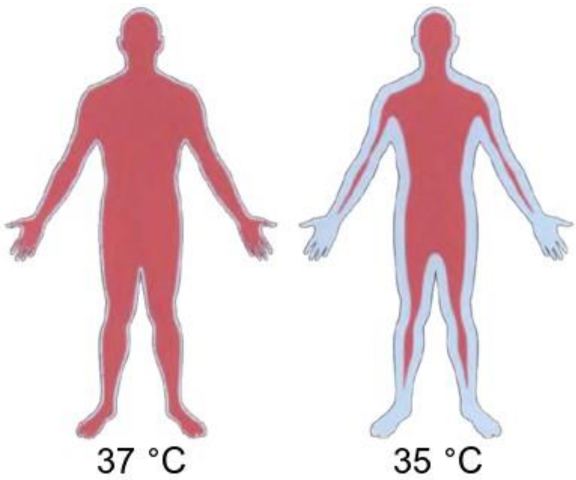

Figure 1: Human-body temperature decrease [2]

Consider a naked human body fully immersed into cold water, such as it might be deep in the ocean. The body loses heat by conduction and convection, while producing heat by means of usual Basal Metabolism Rate (BMR).

The heat equation says that:

$$\rho c \frac{dT}{dt} = k\nabla^2 T + \dddot{Q}_{gen} - \dddot{Q}_{conv}, \qquad (1)$$

where ρ is the human-body density, A the human-body heat capacity, T the body temperature, and k the human-body thermal conductivity. Each term in equation (1) represents heat that is lost or produced per unit volume, thus expressed in W/ m^3. In particular:

$k\nabla^2 T = \dddot{Q}_{cond}$ represents the heat lost by conduction,

$\dddot{Q}_{gen}$ represents the BMR, and

$\dddot{Q}_{conv}$ represents the heat lost by convection.

In the following, equation (1) will be analyzed, in order to discuss the time dependence of the body temperature. The result is expected to be a decreasing function of time, at a rate that depends on water temperature and body properties, which may include, e.g., thermal conductivity, initial temperature of the body, and volume.

Heat equation

Considering heat lost by conduction and by convection and multiplying each one by the volume V, one obtains:[3]

$$\dddot{Q}_{conv}\, V = \dot{Q}_{conv} = \ddot{Q}_{conv}\, A = hA(T - T_{water}), \qquad (2)$$

$$k\nabla^2 TV = \dddot{Q}_{cond}\, V = \dot{Q}_{cond} = \ddot{Q}_{cond}\, A = -kA\frac{dT}{dx}, \qquad (3)$$

where h is the convection coefficient and A is the exposed skin surface of the human body.

Inserting (2) and (3) into equation (1):

$$\rho c V\frac{dT}{dt} = -kA\frac{dT}{dx} + \dot{Q}_{gen} - hA(T - T_{water}) \quad . \qquad (4)$$

Therefore, one has:

$$\frac{dT}{dx} = \frac{(T - T_{water})}{\delta} \quad , \qquad (5)$$

where δ is the skin thickness. Now, using (5) into equation (4) one obtains the equation:

$$\rho c V\frac{dT}{dt} = -kA\frac{(T - T_{water})}{\delta} + \dot{Q}_{gen} - hA(T - T_{water}) \quad . \qquad (6)$$

How does the temperature decrease?

Collecting all the terms in the equation (6) on the left-hand side:

$$\rho c V\frac{dT}{dt} + A\left(\frac{k}{\delta} + h\right)T - A\left(\frac{k}{\delta} + h\right)T_{water} - \dot{Q}_{gen} = 0 .$$

This is a first order non-homogeneous differential equation in the function T(t), with initial condition $T(0) = T_i$, where T_i is the initial body temperature. Using standard analytical methods, the solution is easily found to be:

$$T(t)=\left[T_i - T_{water} - \frac{\dot{Q}_{gen}}{A\left(\frac{k}{\delta}+h\right)}\right]e^{\frac{-A\left(\frac{k}{\delta}+h\right)t}{\rho c V}} + T_{water} + \frac{\dot{Q}_{gen}}{A\left(\frac{k}{\delta}+h\right)}. \quad (7)$$

The time constant with which the temperature decreases is equal to:

$$\tau = \frac{\rho c V}{A\left(\frac{k}{\delta}+h\right)}. \quad (8)$$

The longer is tau, the slower will be the rate at which the temperature lowers. The value of tau is determined by the physical quantities that characterize the human body and water, in a manner that will be discussed in the following concluding remarks.

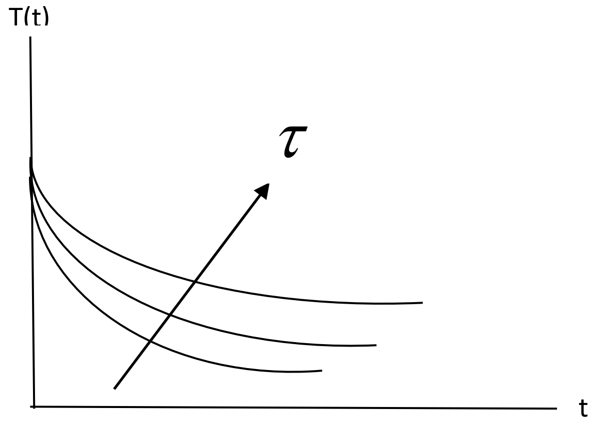

Figure 2: Temperature in function of time

Conclusions

Looking at equation (8), one can first notice that the temperature decreases more slowly for larger-volume bodies with larger heat capacity. The larger the thermal conductivity over to the skin thickness, the convection coefficient and the body surface are, the faster will be the temperature lowering. Since the law is exponential, one can also remark that temperature is expected to decrease much faster at the beginning. Also it is considered δ as skin thickness for reasons of simplification, but also fat thickness [1] has a significant influence on the rate at which the function decreases and therefore on survival time.

References

[1] Heat transfer model for predicting survival time in cold water immersion, F.Tarlochan, S.Ramesh (2005).

[2] www.biologiamarina.eu, tempo di sopravvivenza in mare.

[3] Dispensa di Fenomeni di Trasporto Biologico (2020), A.Ahluwalia, Ingegneria Biomedica Unipi.