Quasi-Static Analysis of Synergistically Underactuated Robotic Hands in Grasping and Manipulation Tasks

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Abstract As described in Chapters 1, 2, 3 and 4, neuroscientific studies showed that the control of the human hand is mainly realized in a *synergistic* way. Recently, taking inspiration from this observation, with the aim of facing the complications consequent to the high number of degrees of freedom, similar approaches have been used for the control of robotic hands. As Chapter 11 describes SynGrasp, a useful technical tool for grasp analysis of synergy-inspired hands, in this Chapter recently developed analysis tools for studying robotic hands equipped with *soft synergy* underactuation (see Chapter 7) are exhaustively described under a theoretical point of view.

After a review of the quasi-static model of the system, the *Fundamental Grasp Matrix* (FGM) and its *canonical form* (cFGM) are presented, from which it is possible to extract relevant information as, for example, the subspaces of the *controllable internal forces*, of the *controllable object displacements* and the *grasp compliance*.

The definitions of some relevant types of manipulation tasks (e.g. the *pure squeeze*, realized maintaining the object configuration fixed but changing contact forces, or the *kinematic grasp displacements*, in which the grasped object can be moved without modifying contact forces) are provided in terms of nullity or non-nullity of the variables describing the system. The feasibility of such predefined tasks can be verified thanks to a decomposition method, based on the search of the *row reduced echelon form* (RREF) of suitable portions of the solution space.

Moreover, a geometric interpretation of the FGM and the possibility to extend the above mentioned methods to the study of robotic hands with different types of

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underactuation are discussed.

Finally, numerical results are presented for a power grasp example, the analysis of which is initially performed for the case of fully-actuated hand, and later verifying, after the introduction of a synergistic underactuation, which capacities of the system are lost, and which other are still present.

1 Introduction

The research in robotic hand design was directed for long time to increase the dexterity and the manipulation capabilities. To follow this line, the number of degrees of freedom, and, more in general, the complexity of the design are increased in the years. Remarkable examples of such hands are the UTAH/MIT hand [1], the Robonaut Hand [2], the Shadow hand [3] and the DLR hand arm system [4], just for citing a few of them, as discussed in Chapter 7.

However, a large number of degrees of freedom, often, bring to enlarge weights and costs of such prototypes. Moreover, the expected advantages in terms of manipulability are often difficult to exploit in a real scenario. Recently, in order to face the complexity of such systems, the human hand was considered as a source of inspiration (see Chapters 1, 7 and 8) not just for the mechanical design, but also in order to simplify the control strategies.

In recent years, many neuroscientific studies such as, for example, the ones discussed in [5], [6], [7], [8], [9], [10], and [11] (see also Chapters 1 - 6), despite significant differences in the definitions and in the requirements of the investigated tasks, share a main observation: simultaneous motion of multiple digits, also called *synergies*, occurs in a consistent fashion, even when the task may require a fairly high degree of movement individuation, such as grasping a small object or typing.

As extensively discussed in the previous Chapters, one of the main result is that a large variety of everyday human grasps is well described by just five synergies. Moreover, the first two human synergies can describe the 80% of the variance in human grasp postures (see also Chapter 8). This suggested the idea to move the description base for grasping, from the joint space to the human-inspired postural synergy space, taking advantage from the *underactuation*. Between the first approach to this idea, we find [12] and [13], that try to implement a synergistic control via software and via hardware, respectively. Despite each one is characterized by its own peculiarities, they share the common characteristics of rigidly controlling the joint movements, via the synergistic underactuation. As discussed in Chapter 7, in the *soft synergies* approach, proposed in [14], a virtual hand is introduced, attracting the real one via a generalized spring, allowing a certain adaptability of the hand during grasps and manipulations tasks. The influence of the synergistic underactuation, in terms of reducing the hand capabilities in object motion and contact force control, is investigated in [15]. Moreover, the contact force optimization problem was faced in [14], considering the limitations imposed by the underactuation.

The present Chapter, mainly based on the results presented in [16], [17] and [18], describes and studies the quasi-static model of a synergistic underactuated hand grasping an object. Considering the results of the above mentioned papers, despite the fact that the analysis is performed in the neighborhood of an equilibrium configuration, also some non local considerations can be done under a more general, nonlinear kineto-static interpretation. More in detail, in Sec. 2 the congruence and the equilibrium equations of the system are presented in quasi-static form. A compliant contact model is introduced between the hand and the object, in order to cope with the static indeterminacy of the contact force distribution problem. Finally, a quasi-static model for the soft synergy underactuation (discussed in Chapter 7) is provided. The treatise is general enough to consider the presence of hand/object contacts also in the internal limbs of the hand. Moreover, the derivative terms of the hand Jacobian and of the grasp matrix are considered, in order to properly take into account the effects of the contact force preload.

Both the presence of internal contacts and of underactuation can greatly affect the capabilities of the hand/object system, in terms of controllable system variations, e.g. limiting the controllability of the forces and/or the object displacements. This problem is faced in Sec. 3 where, after the *Fundamental Grasp Matrix* (FGM) has been defined, its *canonical form* (cFGM) is derived, from which relevant information on the system can be easily obtained, despite the difficulties introduced by the presence of the synergistic underactuation in the model. In fact, as we will discuss in Sec. 3.2, from the cFGM we can obtain information on the *controllable internal forces*, on the *controllable object displacements*, and on the *grasp compliance*, i.e. the compliance perceived at the object level. Moreover, from the cFGM, input-output relationships between the *independent variables* (i.e. the joint displacements and the external wrench variation) and the *dependent variables* of the system can be easily deduced.

In order to go beyond the information provided by the cFGM, a method to investigate the solution space of the system is presented in Sec. 4. Different types of system behaviors are defined in terms of nullity or non-nullity of some system variables, such as, for example, the *pure squeeze*, where the contact forces are modified without affecting the object configuration, or the *kinematic grasp displacement*, where, on the contrary, an object movement is allowed, without changing the contact forces. Finally, a decomposition method, based on the *row reduced echelon form* (RREF) is presented, in order to find out the feasibility of those predefined solutions.

In Sec. 5 a geometrical interpretation of the FGM is given. With a proper arrangement of the equations, the FGM takes the form of a first-order Taylor series approximation of the *equilibrium manifold* (EM) of the whole system, describing the kineto-static behavior both of the hand and of the object during their interaction. As explained in [18], some properties of the EM can be exploited, in order to steer the system, along a trajectory composed by a sequence of equilibrium configurations, toward a final one, characterized by the desired kineto-static properties.

Many of the observations and methods presented can be applied, with small modifications, also in case of different types of underactuation, and the Sec. 6 is dedicated to discuss this topic (see also Chapter 11).

To conclude, in Sec. 7, a numerical example is presented, for a power grasp case. The example was firstly studied as if the hand was completely actuated, discovering its manipulation capabilities. Then, a synergistic underactuation is introduced, and the methods presented in the Chapter are used to verify which possibilities are lost and which others are still present.

2 System Modeling

In this Section, we will present the equations describing the quasi-static behavior of the hand/object system, schematically represented in Fig. 1, and already introduced in Chapters 7 and 11. For both the hand and the object, the quasi-static equilibrium equations will be considered, obtained as a first order approximation of the general, nonlinear, equilibrium equations. Moreover, in connection with the previous, by means of kineto-static duality considerations, the congruence equations will be introduced, describing the displacement of the contact points, corresponding to the hand/object displacements. A linear elastic model for the contact is also introduced, in order to properly describe how the contact forces change, during the execution of a manipulation tasks. Finally, the underactuation will be introduced in the system according to the soft synergy pattern.

For the sake of clarity, in the following we will briefly recall some of the notations already introduced in Chapters 7 and 11, also summarized in Table 1.

2.1 Object Equations

2.2 Equilibrium Equation of the Object

The grasped object is in equilibrium if the sum of all the contact forces/torques exerted by the hand, gathered in the contact force vector[1] $f_c \in \mathbb{R}^c$, and of a possible external wrench[2] $w \in \mathbb{R}^{\sharp w}$ is null, where the symbol $\sharp x$ indicates the dimension of the vector x. In the present discussion, the contact forces are considered to be expressed

[1] The dimension of the contact force vector c is related to the number of contact points and to the local characteristics of the contacts. More details about this will be provided in Sec. 2.5.

[2] Strictly speaking, the vector $w \in \mathbb{R}^{\sharp w}$, in the present dissertation, represents a *parametrization of an external wrench*, abbreviated in the text simply as *external wrench*. Similarly, the object configuration is described by a parametrization vector $u \in \mathbb{R}^{\sharp w}$. As a consequence, the object velocity $\dot{u}$ in (3) is a parametrization of the object twist, and, for this reason, can be expressed as the time derivative of some physical variables. As an example, in a $3D$ case, a complete parametrization can be obtained considering a $6-$DoF virtual kinematic chain describing the configuration of the

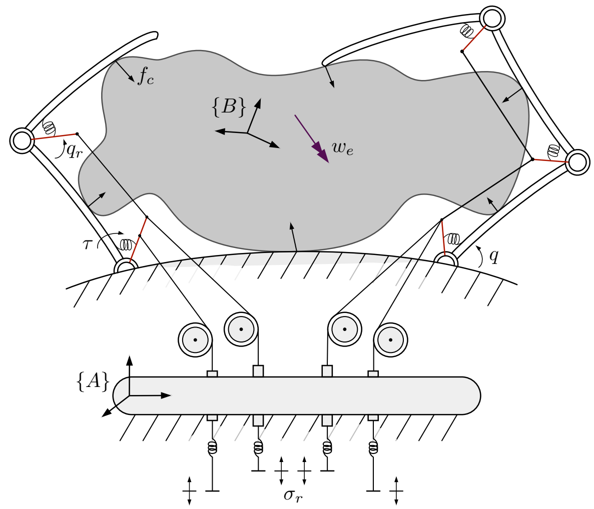

Fig. 1: Reference scheme for the analysis of compliant grasp by synergistically underactuated robotic hand.

in a local frame attached to the object. Before summing all the contributions, they have to be all expressed in a same reference frame, as for example the frame $\{B\}$ in Fig. 1, attached to the object. To this aim, it is usual in literature to introduce the *grasp matrix*, indicated as $G \in \mathbb{R}^{\sharp w \times c}$. Using the previous symbols, the equilibrium law for the object can be written as

$$w + G f_c = 0. \tag{1}$$

It is worth observing that, despite the fact that the contact forces are described in a local frame attached to the object, the parametrization of the external wrench imposes that the grasp matrix becomes a function of the object configuration, as explained in [18]. In light of this, by means of a first-order Taylor series approximation, from (1) the quasi-static equilibrium equation for the object can be obtained in the form

$$\delta w + G \delta f_c + U_g \delta u = 0, \tag{2}$$

where the symbol δx expresses the variation of the variable x, the vector[2] $u \in \mathbb{R}^{\sharp w}$ describes a parametrization of the object configuration and $U_g := \frac{\partial G f_c}{\partial u}$.

object frame with respect to a fixed one. In this case, the vectors $\dot{u}$ and w will contain, respectively, the joint velocities and the joint torques of the virtual kinematic chain.

Notation	Definition
δx	variation of the variable x
$\sharp x$	dimensions of the vector x
$q \in \mathbb{R}^{\sharp q}$	joint configuration
$q_r \in \mathbb{R}^{\sharp q}$	joint reference
$\tau \in \mathbb{R}^{\sharp q}$	joint torque
$\sigma \in \mathbb{R}^{\sharp \sigma}$	synergy configuration
$\sigma_r \in \mathbb{R}^{\sharp \sigma}$	synergy reference
$\eta \in \mathbb{R}^{\sharp \sigma}$	synergy actuation (generalized) force
c	number of hand/object contact constraints
$f_c \in \mathbb{R}^{c}$	contact force/torque vector exerted by the hand on the object
$p_h^o \in \mathbb{R}^{c}$	pose of the hand contact frame with respect to the object contact frame
$w \in \mathbb{R}^{\sharp w}$	(parametrized) external wrench acting on the object; $\sharp w = 6$ in 3D case, $\sharp w = 3$ in planar case
$u \in \mathbb{R}^{\sharp w}$	(parametrized) object frame configuration
$J \in \mathbb{R}^{c \times \sharp q}$	hand Jacobian matrix
$S \in \mathbb{R}^{\sharp q \times \sharp \sigma}$	synergy matrix
$G \in \mathbb{R}^{\sharp w \times c}$	grasp matrix
$\Phi^{\star}$	*Fundamental Grasp Matrix*, the coefficient matrix of the *Fundamental Grasp Equation* (14)
φ	*augmented configuration*, vector collecting the kineto-static variables of the system

Table 1: Notation for grasp analysis.

2.3 Congruence Equation of the Object

From (1), by kineto-static duality considerations, it is possible to find that the transpose of the grasp matrix maps the object velocity[2], indicated as $\dot{u} \in \mathbb{R}^{\sharp w}$, into the velocities of the object contact frames, grouped into the vector $v_o \in \mathbb{R}^{c}$, as follows[3]

$$v_o = G^T \dot{u}. \tag{3}$$

The congruence equation, describing the displacements of the contact frames as a consequence of the object frame displacement, can be obtained from (3) by multiplying each member for an infinitesimal amount of time dt, obtaining

$$\delta C_o = G^T \delta u. \tag{4}$$

[3] More precisely, the vectors v_o and v_h contain the terms of the contact frame twists violating the (rigid) contact constraints between the hand and the object.

2.4 Hand Equations

2.4.1 Congruence Equation of the Hand

Let us define the *hand Jacobian matrix*, $J \in \mathbb{R}^{c \times \sharp q}$, as the map between the joint velocities, clustered in the vector $\dot{q} \in \mathbb{R}^{\sharp q}$, and the velocities of the hand contact frames[3] $v_h \in \mathbb{R}^c$, such that

$$v_h = J\,\dot{q}. \tag{5}$$

The displacement of the contact frames attached to the hand can be obtained by multiplying each member of (5) for an infinitesimal amount of time dt, obtaining

$$\delta C_h = J \delta q, \tag{6}$$

that describes the quasi-static form of the congruence equation of the hand.

2.4.2 Equilibrium Equation of the Hand

The equilibrium law for the hand comes from (5) by kineto-static duality considerations. As a result, indicating with the symbol $\tau \in \mathbb{R}^{\sharp q}$ the joint torque vector, the equilibrium law for the hand can be expressed as

$$\tau = J^T f_c. \tag{7}$$

The quasi-static equilibrium equation is obtained from (7), by means of a first order Taylor series expansion. To this aim, it is important to note that, since the fact that the contact forces are described in a local frame attached to the object, the Jacobian matrix, introduced in (5), is a function both of the joint parameters of the hand q, and of the object configuration parameters u, that is $J = J(q,u)$.

From these considerations, it follows that the quasi-static equilibrium of the hand can be expressed as

$$\delta \tau = Q_j \delta q + U_j \delta u + J^T \delta f_c, \tag{8}$$

where $Q_j := \frac{\partial J^T f_c}{\partial q}$ and $U_j := \frac{\partial J^T f_c}{\partial u}$.

2.5 Hand/Object Interaction Model

In the contact between the hand and the object, relative displacements of the contact frames are forbidden in some directions. In these directions, some reaction forces can arise. The dimension c_i of the i^{th} reaction force vector depends by the nature of the materials involved. As an example, in the case of *contact point with friction*, or *hard finger* contact model, the force can be transmitted in any direction, but no moment is allowed, that is $c_i = 3$. Indeed, in the case of *soft-finger* contact type, also a moment about the normal to the contact can be transmitted, thus $c_i = 4$.

In most cases of interest, the total number of contact force elements $c = \sum_i c_i$ is greater than the number of the external wrench elements. For this reason, the problem of determining the contact force distribution is statically indeterminate.

This problem is generally faced in literature by *relaxing* the contact constraints. In other words, a relative displacement of the contact frames is allowed also in the directions nominally forbidden by the (rigid) contact constraint, and this is interpreted as the cause of the contact force variation. This behavior is modeled introducing a (virtual) linear spring between the two bodies in contact. Defining $K_c \in \mathbb{R}^{c \times c}$ as the contact stiffness matrix, i.e. a matrix collecting the stiffness values of all the contact springs, the constitutive equation of the contact can be, finally, expressed as

$$\delta f_c = K_c(\delta C_h - \delta C_o). \tag{9}$$

2.6 Soft Synergy Underactuation Model

As explained in Sec. 1, in this Chapter we consider the problem of discovering the capabilities of *soft synergy* underactuated robotic hands in grasping, as already discussed in Chapter 7. Inspired by neuroscientific studies, the *soft synergy* underactuation model, can be seen as composed by two elements: (i) a virtual hand, which movement is governed by a synergistic correlation of the joints, and (ii) a set of virtual springs, connecting the virtual hand to the real one.

To mathematically describe this model, in each joint we introduce a compliant element by means of which the joint reference variables, collected in the vector $q_r \in \mathbb{R}^{\sharp q}$, transmit the motion to the real ones. Afterwards, the synergistic behavior of the hand is obtained imposing a correlation between the joint reference variables.

2.6.1 Elastic Joint Model

The equilibrium condition for the elastic joints requires that joint torques and the spring deflections, that is the mismatch between the reference joint variables and the real ones, are related by the joint stiffness. Considering this, by the introduction of the joint stiffness matrix $K_q \in \mathbb{R}^{\sharp q \times \sharp q}$, collecting all the joint stiffness values, it directly follows that the quasi-static equilibrium law for the elastic joints is described by the following

$$\delta \tau = K_q(\delta q_r - \delta q). \tag{10}$$

2.6.2 Introducing Synergies

The synergistic underactuation is imposed to the system by means of the *synergy matrix* $S \in \mathbb{R}^{\sharp q \times \sharp \sigma}$. In analogy to what seen in (6), the joint reference displacements can be expressed as

$$\delta q_r = S \delta \sigma, \tag{11}$$

where $\sigma \in \mathbb{R}^{\sharp \sigma}$ is the synergistic actuation vector.

Again, by virtue of the kineto-static duality, indicating with $\eta \in \mathbb{R}^{\sharp \sigma}$ the generalized actuation forces at the synergy level, with considerations similar to those that have led to (8), the quasi-static equilibrium for the synergistic underactuation level can be written as

$$\delta \eta = S^T \delta \tau + \Sigma \delta \sigma, \tag{12}$$

where $\Sigma := \frac{\partial S^T \tau}{\partial \sigma}$.

As already seen for the joints, an elastic model can also be introduced for the synergistic actuation by means of a synergy reference variable $\sigma_r \in \mathbb{R}^{\sharp \sigma}$, and the *synergy stiffness matrix* $K_\sigma \in \mathbb{R}^{\sharp \sigma \times \sharp \sigma}$. Thus, similarly to what seen in (10), the elastic actuation model for the synergy actuation can be described as

$$\delta \eta = K_\sigma (\delta \sigma_r - \delta \sigma). \tag{13}$$

2.7 *The Fundamental Grasp Equation*

Grouping together the equations for the object, the hand and the synergistic underactuation, that is considering the eq.s (2), (4), (6), (8), (9), (10), (11), (12), (13), denoting with I an identity matrix of proper dimensions, we obtain the system

$$\begin{bmatrix} G & 0 & 0 & U_g & 0 & 0 & I & 0 \\ -J^T & I & 0 & -U_j & -Q_j & 0 & 0 & 0 \\ I & 0 & 0 & K_c G^T & -K_c J & 0 & 0 & 0 \\ 0 & I & 0 & 0 & K_q & -K_q S & 0 & 0 \\ 0 & -S^T & I & 0 & 0 & -\Sigma & 0 & 0 \\ 0 & 0 & I & 0 & 0 & K_\sigma & 0 & -K_\sigma \end{bmatrix} \begin{bmatrix} \delta f_c \\ \delta \tau \\ \delta \eta \\ \delta u \\ \delta q \\ \delta \sigma \\ \delta w \\ \delta \sigma_r \end{bmatrix} = 0, \tag{14}$$

where the contribution of (4) and (6) was considered in (9), as well as (11) was considered in (10).

Eq. (14), also called *Fundamental Grasp Equation* (FGE), is a *linear* and *homogeneous* system, that can be written in compact form as $\Phi^\star \delta \varphi = 0$. The coefficient matrix of the system, $\Phi^\star \in \mathbb{R}^{r_\Phi \times c_\Phi}$ is the *Fundamental Grasp Matrix* (FGM), which matrix elements are evaluated in the reference equilibrium configuration of the system, and the variable vector $\delta\varphi \in \mathbb{R}^{c_\Phi}$ is the *augmented configuration*, collecting the variation of the system variables.

By direct inspection of (14), it is easy to verify that for the number of rows and columns of the FGM, that is for r_Φ and c_Φ respectively, it holds that

$$\begin{aligned} r_\Phi &= \sharp f_c + 2\sharp q + 2\sharp\sigma + \sharp w, \\ c_\Phi &= \sharp f_c + 2\sharp q + 3\sharp\sigma + 2\sharp w. \end{aligned} \tag{15}$$

In most cases of practical relevance the FGM is full row rank[4], that is $\mathrm{rank}(\Phi^\star) = r_\Phi$, and we will assume it in the rest of the dissertation. In these cases, eq. (14) can be univocally solved when it is known a number of *independent variables*, or *inputs* for the system, equal to $c_\Phi - r_\Phi = \sharp w + \sharp\sigma$. In continuity with the grasp analysis literature, we consider to known, or to have a measure of, the external wrench variation δw. Moreover, the synergy references are supposed to be position-controlled, thus we consider to know[5] the variable $\delta\sigma_r$. The independent variables will be jointly indicates in next sections as $\delta\varphi_i \in \mathbb{R}^{c_\Phi - r_\Phi}$. We will refer to the set of all the other variables as the *dependent variables*, or *output* of the system, and they will be indicated as $\delta\varphi_d \in \mathbb{R}^{r_\Phi}$.

3 Controllable System Configuration Variations

3.1 The Canonical Form of the Fundamental Grasp Equation

Considering previous definitions, eq. (14) can be also written as

$$\Phi^\star\delta\varphi = \begin{bmatrix} \Phi_d^\star & \Phi_i^\star \end{bmatrix} \begin{bmatrix} \delta\varphi_d \\ \delta\varphi_i \end{bmatrix} = 0. \tag{16}$$

Assuming the invertibility[4] of the matrix $\Phi_d^\star$, the so called *canonical form of the Fundamental Grasp Equation* (cFGE) can be obtained left-multiplying (16) for ${\Phi_d^\star}^{-1}$, thus obtaining

$$\begin{bmatrix} I & \Phi_i \end{bmatrix} \begin{bmatrix} \delta\varphi_d \\ \delta\varphi_i \end{bmatrix} = 0, \tag{17}$$

where $\Phi_i = {\Phi_d^\star}^{-1}\Phi_i^\star$. It is worth observing in passing that, since the matrix ${\Phi_d^\star}^{-1}$ is full rank, eq.s (16) and (17) have the same solution space. In other words, all the vectors $\delta\varphi$ satisfying (16) are also a solution of (17).

[4] Exceptions are analytically possible but they refer to pathological situations of poor practical interest.

[5] Other choices are possible, as for example considering to know the object displacement δu, instead of the external wrench δw, or the actuation force variation $\delta\eta$, instead of the synergistic displacement variable $\delta\sigma_r$. Many results of our analysis can be easily adapted to the above mentioned situations as well.

The coefficient matrix of (17), characterized by the presence of an identity block corresponding to the dependent variables, is the *canonical form of the fundamental grasp matrix* (cFGM). From (17), it is easy to find that, once the variation of the independent variables is known, the value of the dependent variable variation can be directly computed as

$$\delta\varphi_d = -\Phi_i \delta\varphi_i, \tag{18}$$

which represents, in compact form, the raltionship between the input and the output variables of the system.

3.2 *Relevant Properties of the Canonical Form of the Fundamental Grasp Matrix*

The cFGM can be further investigated, in order to find out some relevant information on the characteristics of the physical system. To this aim, let us consider again (17). More in detail, this can be written also as

$$\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & W_f & R_f \\ 0 & I & 0 & 0 & 0 & 0 & W_\tau & R_\tau \\ 0 & 0 & I & 0 & 0 & 0 & W_\eta & R_\eta \\ 0 & 0 & 0 & I & 0 & 0 & W_u & R_u \\ 0 & 0 & 0 & 0 & I & 0 & W_q & R_q \\ 0 & 0 & 0 & 0 & 0 & I & W_\sigma & R_\sigma \end{bmatrix} \begin{bmatrix} \delta f_c \\ \delta\tau \\ \delta\eta \\ \delta u \\ \delta q \\ \delta\sigma \\ \delta w \\ \delta\sigma_r \end{bmatrix} = 0. \tag{19}$$

3.2.1 Controllable Internal Forces

From (19), we can extract the expression for the contact force variation, that is

$$\delta f_c + W_f \delta w + R_f \delta\sigma_r = 0. \tag{20}$$

In continuity with the literature, we define as *internal* the solutions of (19), or equivalently of (14), not involving the external wrench variation. From this definition, it immediately follows that the matrix R_f spans the subspace of the *controllable internal forces*, that is the subset of all the contact force variations that can be generated controlling the synergistic movement of the hand.

3.2.2 Contact Force Transmission Caused by an External Wrench

Again from (20), considering the hand actuation kept constant, the matrix W_f represents a map between the external wrench and the contact force variation.

In other words, $-W_f$ represents the *contact force transmission* caused by an external wrench variation.

Both controllable internal forces and contact force transmission have great relevance in some grasping problems, as e.g. in the force closure evaluation and in the contact force optimization problem.

3.2.3 Controllable Internal Object Displacements

In case of whole-hand grasp and/or of underactuated hands, it could be not easy to find out which motions can be imposed to the grasped object by the hand. The problem can be solved considering the fourth equation of (19), that provides a description of the object displacements as

$$\delta u + W_u \delta w + R_u \delta \sigma_r = 0. \tag{21}$$

Similarly to what discussed in Sec. 3.2.1, from (21) we can easily conclude that the matrix R_u spans the subspace of the *controllable internal object displacements*.

3.2.4 Grasp Compliance

Again from (21), we can find that the matrix $-W_u$ represents the *grasp compliance*. In other words, the matrix $C_g = -W_u$ is the compliance that a 6D spring should have in order to imitate the effects of the hand actuation on the object displacements, when an external wrench is applied.

3.3 GEROME-B: a Specialized Gauss Elimination Method for Block Partitioned Matrices

In Sec. 3.1, a numerical method to compute the cFGM was presented. Furthermore, the physical interpretation of some blocks composing the cFGM was discussed, providing relevant information on the hand/object system. However, since the relevance of these blocks, it may be helpful to have a symbolic form of the matrices W_j and R_j in (19), in order to better understand how some basic matrices of the system (such as the Jacobian matrix J, the grasp matrix G, the synergy matrix S, etc...) can affect the properties of the whole system (e.g. the controllable internal forces or the controllable displacement of the object). Moreover, the knowledge of such symbolic relationships can be profitably used e.g. in designing robotic hands or underactuation mechanism. An example can be found in Chapter 7, regarding the design of the underactuation of the Pisa/IIT SoftHand.

To achieve this goal, the typical *Gauss Elementary Row Operation Method* (GEROME) for linear and homogeneous systems was adapted to act on block partitioned matrices (GEROME-B), preserving the integrity of the initial blocks (see also Chapter 7).

The GEROME-B method can be applied by means of the following three elementary operations:

- exchanging the i^{th} row-block with the j^{th} row-block
- multiplying the i^{th} row-block by a full-rank matrix Δ,
- adding the i^{th} row-block with the j^{th} row-block, possibly left-multiplied for a suitable matrix Λ to accord dimensions.

Each rule can be performed by left-multiplying the FGM for a suitable full-column rank matrix, thus without affecting the solution space of the initial system.

Let us consider a proper identity matrix I_p, initially partitioned such that the i^{th} block on the main diagonal, indicated as I_{p_i}, has the same dimensions of the i^{th} row-block of the FGM. From this, the three matrices, equivalent to the three elementary operations previously seen, can be written as

$$\begin{aligned}
&M^1_{ij} = \text{diag}(I_{p_1},\dots,I_{p_{i-1}},I_{p_j},I_{p_{i+1}},\dots,I_{p_{j-1}},I_{p_i},I_{p_{j+1}},\dots,I_{p_m}),\\
&M^2_{ii}(\Delta) = \text{diag}(I_{p_1},\dots,I_{p_{i-1}},\Delta,I_{p_{i+1}},\dots I_{p_m}),\\
&M^3_{ij}(\Lambda) = I_p \oplus \Lambda_{ij},
\end{aligned} \tag{22}$$

where the expression $I_p \oplus \Lambda_{ij}$ indicates the insertion of a suitable matrix Λ on the block on the i^{th} row and j^{th} column of the default partitioned identity matrix I_p, and where m is the number of row-blocks of the identity matrix I_p.

Moreover, similarly to the classical elimination method, to apply GEROME-B it is necessary to define and identify some *pivot* elements.

Definition 1. A block of the FGM can be a *pivot* if

- it is a full-rank square block,
- it is the only pivot in its row and column,
- it is not a coefficient of one of the input variables.

Without losing generality, describing the algorithm, we suppose to act on a matrix $\hat{\Phi}^\star$, such that all the pivots are on the main diagonal. The matrix $\hat{\Phi}^\star$ can be obtained from the initial $\Phi^\star$ by properly exchanging some rows and columns and/or using matrices of the type (22). Once the algorithm is completed, if desired, the permutation can be inverted, restoring the initial order. In our case, the desired new form of the FGM can be written as

$$\hat{\Phi}^\star = \begin{bmatrix}
I & 0 & 0 & K_c G^T & -K_c J & 0 & 0 & 0\\
-J^T & I & 0 & -U_j & -Q_j & 0 & 0 & 0\\
0 & -S^T & I & 0 & 0 & -\Sigma & 0 & 0\\
0 & 0 & 0 & U_g - G K_c G^T & G K_c J & 0 & I & 0\\
0 & I & 0 & 0 & K_q & -K_q S & 0 & 0\\
0 & 0 & I & 0 & 0 & K_\sigma & 0 & -K_\sigma
\end{bmatrix}. \tag{23}$$

The three matrices seen in (22) can be used to describe the GEROME-B algorithm, able to bring to the cFGM acting on the new form of the coefficient matrix (23). The GEROME-B algorithm essentially operates through the following steps: (i) the i^{th} block row is left-multiplied for the inverse of the i^{th} pivot, thus the i^{th} pivot becomes an identity matrix; (ii) the i^{th} pivot is used to cancel out all the elements on its same column; (iii) the process is iterated for all the pivots. A formal description of these steps is presented in Algorithm 1.

Algorithm 1 GEROME-B

```
for h = 1 → m do
    Δ = Φ̂*_hh^{-1}
    Φ̂* = M²_hh(Δ)Φ̂*
  for k = 1 → m do
    if h ≠ k then
      Λ = −Φ̂*_kh
      Φ̂* = M³_kh(Λ)Φ̂*
    end if
  end for
end for
```

4 Solution Space Decomposition

Among all the possible solutions of the system, several are of greater practical interest. As a simple example, let us consider an object placement task. During the motion of the object, uncertainties of the model, as well as external disturbances, could bring one or more contacts close to the slipping condition. In order to increase the robustness of the grasp without affecting the performances of the positioning task, it is important to recognize the capability of the hand of redistributing internal forces, avoiding object movements. From this and other simple examples, it follows that some interesting behavior of the system can be described by defining proper (non-)nullity patterns of the system variables. In this way, in this Section, some particular types of solutions will be defined, together with a method to discover their feasibility, by means of a numerical procedure acting on the solution space of the system, that is on the nullspace of the FGM.

4.1 Relevant Types of System Solutions

4.1.1 Internal System Perturbations

As discussed in Sec. 3.2.1, following the grasping literature, we will call *internal* the solutions in which an external wrench variation does not appear, that is in the cases in which $\delta w = 0$.

4.1.2 Pure Squeeze

We define the *pure squeeze* as the particular system behavior in which there is a contact force variation not caused by an external wrench, and do not involving any object displacements. In other words, a pure squeeze occurs if $\delta w = 0, \delta f_c \neq 0$ and $\delta u = 0$.

4.1.3 Spurious Squeeze

An internal contact force redistribution associated to a displacement of the object is defined as *spurious squeeze*. The definition correspond to a solution of the form $\delta w = 0, \delta f_c \neq 0$ and $\delta u \neq 0$.

4.1.4 Kinematic Grasp Displacement

The internal solutions in which the object is moved without changing the contact force distribution, that is do not violating the (rigid) kinematic contact constraints, are called *kinematic grasp displacement*. Such solutions have to verify the conditions $\delta w = 0, \delta f_c = 0$ and $\delta u \neq 0$.

It is worth observing that, considering the elastic model of the contact as descriptive of the deformations of the grasped object, requiring a null variation of contact forces implies a null variation of the object shape. In this interpretation the definition of *rigid object displacement* can be recovered.

4.1.5 External Structural Force

An *external* action causing a contact force variation without affecting the hand actuation level is defined as *external structural force*. If such kind of solution is possible, it is characterized by $\delta w \neq 0, \delta f_c \neq 0$ and $\delta \eta = 0, \delta \sigma_r = 0$. Considering eq. (13), above conditions directly imply also that $\delta \sigma = 0$.

4.2 Discovering (Non-)Nullity Patterns in the Solution Space

In previous Sections we showed how some relevant types of manipulation tasks can be defined in terms of nullity or non-nullity of some system variables. The feasibility of such solutions can be investigate by properly elaborating the solution space of the FGM. In this Section, we briefly present a method to discover if the hand/object system is able to perform a task corresponding to a solution of (14), in the desired form. To this aim, we firstly recall some results from linear algebra, the details of which can be found in [19]. For the following discussion, it is useful to recall that from every matrix $C \in \mathbb{R}^{r_c \times c_c}$, with $\rho_c = \mathrm{rank}(C)$, its corresponding *reduced row echelon form* (RREF) can be obtained via a Gauss-Jordan elimination. The same result can be equivalently obtained by a suitable permutation matrix $\Pi \in \mathbb{R}^{r_c \times r_c}$, such that

$$\Pi C = \left[\frac{U}{0}\right], \tag{24}$$

where $U \in \mathbb{R}^{\rho_c \times c_c}$ is a staircase matrix, and the zero block has consequent dimensions. The RREF of a matrix, in (24), can be profitably used to discover the presence of desired (non-)nullity pattern in the nullspace base $\Gamma \in \mathbb{R}^{r_\gamma \times c_\gamma}$, that is in the solution space of (14). In later discussion, we will make the assumption to have access to a function $\texttt{rref}(X)$ able to return the reduced row echelon form of its argument[6] X.

For the sake of simplicity, we consider the system variables divided in two groups, called $\delta\varphi_\alpha$ and $\delta\varphi_\beta$, and we will present the investigation method supposing that we are interested to find the solutions characterized by $\delta\varphi_\beta = 0$. In this case, all the solutions of the system can be written as

$$\delta\varphi = \begin{bmatrix} \delta\varphi_\alpha \\ \delta\varphi_\beta \end{bmatrix} = \begin{bmatrix} \Gamma_\alpha \\ \Gamma_\beta \end{bmatrix} x, \tag{25}$$

where $\Gamma_\alpha \in \mathbb{R}^{r_\alpha \times c_\gamma}$ and $\Gamma_\beta \in \mathbb{R}^{r_\beta \times c_\gamma}$, the portions of the nullspace relative to the variables just defined.

Considering (25), a suitable permutation matrix can be obtained running the function $\texttt{rref}([\Gamma_\beta^T \mid I])$, which result is a matrix in the form $\left[\begin{array}{c|c} U_\beta & \multirow{2}{*}{Π_β} \\ \hline 0 & \end{array}\right]$, where $U_\beta \in \mathbb{R}^{\rho_\beta \times r_\beta}$, and $\rho_\beta = \mathrm{rank}(\Gamma_\beta)$. From the properties of the RREF, it is known that the block $\Pi_\beta \in \mathbb{R}^{c_\gamma \times c_\gamma}$ is the permutation matrix such that $\Pi_\beta \Gamma_\beta^T = U_\beta$. Using these results, it is possible to find a new form ${}^1\Gamma \in \mathbb{R}^{r_\gamma \times c_\gamma}$ for the solution space matrix such that

$$ {}^1\Gamma = \Gamma \Pi_\beta^T = \left[\begin{array}{c} {}^1\Gamma_\alpha \\ \hline U_\beta^T \mid 0 \end{array}\right], \tag{26}$$

where ${}^1\Gamma_\alpha = \Gamma_\alpha \Pi_\beta^T$. From direct inspection of (26), it is evident that the last $c_\gamma - \rho_\beta$ columns of Γ_1 span all the solutions in which $\delta\varphi_\beta = 0$, while the first ρ_β columns of Γ_1 span all the solutions in which $\delta\varphi_\beta \neq 0$. The method explained can be easily extended, by a recursive application, to the case of searching (non-)nullity conditions for more than one variable. The reader can find more details about the above method in [20] and in [17].

[6] This is a typical situation with the most popular computational platforms, e.g.: $\texttt{rref}(X)$ in MATLAB and `RowReduce(X)` in Mathematica.

5 Geometrical Interpretation of the Fundamental Grasp Equation

In Sec. 2, a model describing the local behavior of a grasp with a synergistic underactuated robotic hand was obtained, starting from both the differential kinematic and the equilibrium equations of the system. The quasi-static form of such equations was obtained considering the effects of the differential kinematic equations for an infinitesimal amount of time, and by means of a first-order Taylor series approximation of the equilibrium equations. Moreover, the constitutive equations of the contacts, as well as the compliance in the actuation (at different levels), were introduced via linear elastic models. All these equations were used to build the *Fundamental Grasp Equation*.

As we saw in (14), it is straightforward considering the contribution of the congruence equations into the other relationships. As a result, eq. (14) can be seen as the first-order approximations of a suitable system of nonlinear equation. Without going into the details, we just mention that such system of equations, the Taylor series approximation of which correspond to eq. (14), can be written as

$$\begin{cases} w + G(u) f_c &= 0 \\ \tau - J^T(q,u) f_c &= 0 \\ f_c - K_c p_h^o &= 0 \\ \tau - K_q(\psi(\sigma) - q) &= 0 \\ \eta - S^T(\sigma)\tau &= 0 \\ \eta - K_\sigma(\sigma_r - \sigma) &= 0, \end{cases} \tag{27}$$

where $p_h^o \in \mathbb{R}^c$ is a vector describing the configuration of the hand contact frames with respect to object ones, and where we introduce the function $\psi(\sigma) := q_r$, such that $\frac{\partial \psi(\sigma)}{\partial \sigma} = S(\sigma)$. We will refer to eq. (27) as the *equilibrium manifold*[7] of the system. We note in passing that the FGE is the equation of the hyperplane tangent to the equilibrium manifold in a specific point, representing an equilibrium configuration of the system.

It is worth observing that, given the invertibility of the matrix $\Phi_d^\star$ in (16), the variables δq and δw can be considered a local parametrization of the equilibrium manifold in the neighborhood of a given equilibrium configuration of the system. As discussed more in detail in [18], this property can be exploited in order to steer the system toward a new equilibrium configuration characterized by different kineto-static properties, with respect to the initial one. Moreover, as explained in [21], the equilibrium manifold of the system can be used as the exploration space for planning algorithm for closed kinematic chains as e.g. in bimanual manipulation tasks, taking advantage of the compliance in the contacts for *relaxing* the geometric constraints imposed by the presence of the closed loop. In this case, the above

[7] More precisely, the equations related to the elasticity do not describe an equilibrium law, and, for this reason, we should, more properly, talk about a *manifold describing the kineto-static behavior of the whole system*. For the sake of compactness, this definition will be left implicit in the rest of the discussion.

discussed equilibrium manifold can be used for random sampling based technique in order to generate any-time paths for closed-loop robot manipulators.

6 Other Types of (Under-)Actuation

Despite the fact that the *soft synergy* (Chapter 7) is currently one of the most attractive and interesting underactuation approach, it is worth considering the possibility to apply the analytical tools presented in this Chapter also in other cases. In literature, other underactuation approaches deserve attention, as e.g. the *eigengrasp*, presented in [12], the parallel structure based [22], or the recent *adaptive synergies* approach, described in [23] and in Chapter 7. Some parts of the previous discussions were strictly dedicated to the *soft synergy underactuation*, especially in Sec. 2. However, the methods presented in Sec. 3 and in Sec. 4 can be easily recovered for other types of underactuation (as also discussed for the methods in Chapter 11). After the kinematic and static equations were obtained in quasi-static form for the particular underactuation mechanism in exam, the *Fundamental Grasp Matrix* directly follows. From this, a proper definition of the *dependent* and the *independent* variables bring to obtain the FGM in *canonical form*. Moreover, the GEROME-B algorithm can still be applied, obtaining the symbolic form of the block matrix composing the cFGM. These results can be used to study how the underactuation affects the main system characteristics. Many definitions of manipulation tasks by (non-)nullity patterns can be recovered, regardless of the particular type of underactuation. One remarkable exception is the subspace of the *external structural forces*. However, the definition provided in Sec. 4.1.5 can be generalized considering the conditions $\delta w \neq 0, \delta f_c \neq 0$, and $\delta\tau^\star = 0, \delta q^\star = 0$, where $\delta q^\star$ and $\delta\tau^\star$ are the generalized displacement and force variables at the underactuation level.

In Chapter 7, more space is dedicated to the application of some of the discussed methods to the case of the *adaptive synergies* undearctuation model.

7 Numerical Results

7.1 Power Grasp

As a test case, we consider a spider-like hand, composed by two fingers and 8 joints, grasping a square of side $2L$. Fig. 2 shows the initial configuration of the system and the contact force preload. All the initial force components have unitary value along the directions depicted.

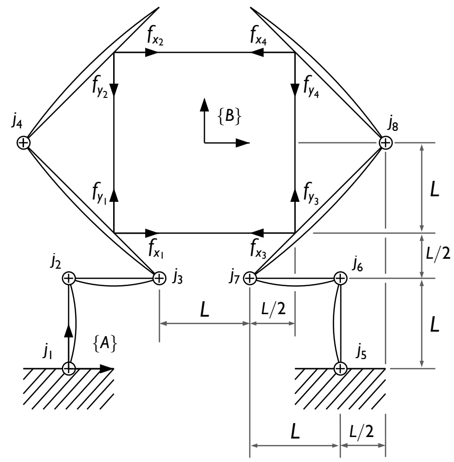

Fig. 2: Compliant grasp of a square object by a two fingered spider-like hand.

7.1.1 Perturbed Configuration for Fully Actuated Hand

The solution space of the system has dimension equal to $\sharp w + \sharp q = 11$. Elaborating the nullspace of the FGM, it is possible to find out that the *pure squeeze* subspace has dimension 5, the *kinematic grasp* subspace has dimension 3 and together they complete the internal solution subspace.

For the *kinematic grasp displacements*, simulation results show that it is possible to have a finite displacement of the object $\delta u_x = 0.001$, as in Fig. 3a, with no torque variations, but with the following joint angle displacements

$$\delta q = 10^{-3}\left[-1\ 1\ 0\ 0\ -1\ 1\ 0\ 0\right]^T. \tag{28}$$

For $\delta u_y = -0.001$, represented in Fig. 3b, the corresponding joint torques and joint angle variations are

$$\begin{aligned}\delta\tau &= 10^{-3}\left[-2\ -2\ 0\ 0\ 2\ 2\ 0\ 0\right]^T,\\ \delta q &= 10^{-3}\left[0\ 1\ -1\ 0\ 0\ -1\ 1\ 0\right]^T.\end{aligned} \tag{29}$$

To obtain an object rotation $\delta u_\alpha = 0.001$, without changing the contact forces, Fig. 3c, the necessary variations in the joint torques and joint angles are

$$\begin{aligned}\delta\tau &= 10^{-3}\left[3\ 3\ 0\ 0\ 3\ 3\ 0\ 0\right]^T,\\ \delta q &= 10^{-3}\left[-1.5 1 1.5 0 -1.5 1 1.5 0\right]^T.\end{aligned} \tag{30}$$

A basis for the pure squeeze is sketched in Fig. 3d, where the couple of forces s_i and $-s_i$ corresponds to the i^{th} components of the basis. The numerical results for $\delta\tau$ and δq are omitted here for brevity.

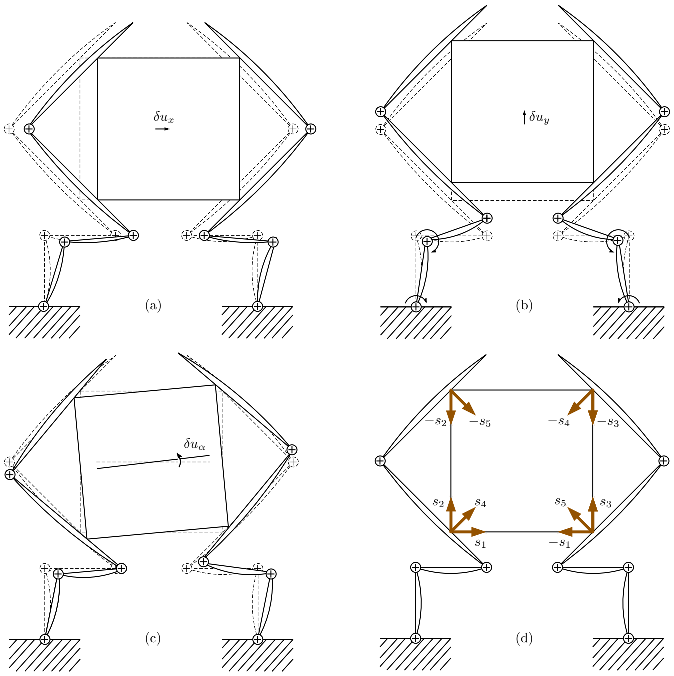

Fig. 3: Plates (a-c) represent the *kinematic displacements* of the grasped object, and plate (d) represents a basis for the *pure squeeze*.

7.1.2 A Synergy in the Power Grasp

Introducing in the system an underactuation characterized by a synergy matrix in the form

$$S = \begin{matrix}[-0.6500 \; 0 \; -0.3200 \;\; -0.4000 \\ 0.6500 \;\;\; 0 \;\;\; 0.3200 \;\;\; 0.4000]^T,\end{matrix} \tag{31}$$

in the solution space it remains a *pure squeeze* subspace of dimension 1.

In the absence of external disturbances, with an unitary synergistic actuation, $\delta\sigma_r = 1$, the contact forces and the object displacements become

$$\delta f_c = \begin{matrix}[0.5043 \;\; 0.5043 \;\; 0.5043 \;\;\;\; -0.5043 \\ -0.5043 \; 0.5043 \; -0.5043 \; -0.5043]^T,\end{matrix} \tag{32}$$

$$\delta u = \begin{bmatrix}0 \; 0 \; 0\end{bmatrix}^T, \tag{33}$$

indicating that we are squeezing the object along both diagonals. It is worth noting that the above synergy was constructed by considering the contribution of two particular *pure squeeze* solutions, represented in Fig. 3d, for the fully-actuated system.

8 Conclusions

In this Chapter, the basic concepts and methods for the quasi-static analysis of synergistically underactuated robotic hands were described. Moreover, compliance was integrated in the system at various levels, i.e. in the contacts between the hand and the object, and in the actuation mechanism, as discussed in Chapter 7. The derivative terms of the hand Jacobian and of the grasp matrix were also considered in the model, in order to properly take into account the effects of the contact force preload. Afterwards, the *Fundamental Grasp Matrix* (FGM) was defined, and a method for finding its *canonical form* (cFGM) was presented, both via a numerical and a symbolic approach. From the cFGM, relevant information on the system behavior can be easily extracted, as e.g. the *controllable internal forces*, the *controllable object displacements* and the *grasp compliance*.

Moreover, a method to investigate the solution space of the FGM was presented, able to point out the feasibility of relevant manipulation tasks, defined in terms of nullity or non-nullity of some system variables.

Despite the fact that the methods proposed provide information about local characteristics of the system around the initial equilibrium configuration, some results have also non-local relevance. In fact, it is possible to provide a geometrical interpretation of the FGE, for which this represents the tangential plane to the *equilibrium manifold* of the whole system. Exploiting the properties of the FGM, a local parametrization of the system can be found, which can be profitably used to steer the system over a continuum set of equilibrium configurations, until the desired kineto-static characteristics were fulfilled.

The generality of the proposed methods, as well as the technical tools described in Chapter 11, can be applied also in case of different types of underactuation, with small modifications.

Finally, in order to assess the validity of the proposed methods, an example of a power grasp has been presented showing the generality of the methods, capable of treating both the cases of fully actuated and synergistically controlled hands.

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