Conflict Resolution Problems for Air Traffic Management Systems Solved with Mixed Integer Programming

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Abstract— **This paper considers the problem of solving conflicts arising among several aircraft that are assumed to move in a shared airspace. Aircraft can not get closer to each other than a given safety distance in order to avoid possible conflicts between different airplanes. For such system of multiple aircraft, we consider the paths planning problem among given waypoints avoiding all possible conflicts. In particular we are interested in optimal path, i.e. we want to minimize the total flight time. We propose two different formulations of the multi-aircraft conflict avoidance problem as a mixed-integer linear program: in the first case only velocity changes are admissible maneuvers, in the second one only heading angle changes are allowed. Due to the linear formulation of the two problems, solutions may be obtained within seconds with standard optimization software, then those approaches may be used as part of a real or fast-time simulation.**

I. Introduction

The current enroute air traffic control system consists for the most part of a geographical network in which aircraft are allowed to fly only along fixed routes. The safety of this architecture is supported by many decades of operations. Under this architecture, the dynamics of the air transportation system is dominated by its network structure, but the increasing demand for air transportation is progressively bringing the entire system to an overloaded and congested state. On the other hand, the continuing improvement of aircraft instrumentation and communication systems carries the potential of resolving these problems via new air traffic control such as the free-flight concept of operations.

Relatively recently, airlines and the Federal Aviation Administration (FAA) have proposed "Free Flight" [1], [2] as a concept of operations relying upon improved communication, navigation and surveillance technology to increase pilot and airline freedom. For example, each pilot would be able to optimize its own trajectory, to minimize the time of flight or to avoid zones of severe weather.

However, the impact of Free Flight upon system safety, as well as the relation between unstructured aircraft flows in Free Flight and air traffic flow management constraints remains largely unknown. To gain some understanding about Free Flight's safety and efficiency requires building fast simulation environments incorporating automated and optimal conflict detection and resolution schemes.

Many approaches have been proposed in the last few years to address the conflict resolution problem when many aircraft are involved; a complete overview of these approaches with a complete bibliography may be found in [5]. For an extensive study on the impact of Free Flight on safety we refer the reader to the work developed at NASA Ames by Bilimoria [13], in which is proved that the Free Flight environment is safer for the current traffic in terms of possible conflict respect to the current airspace structure (see also [4]).

The approach proposed in this paper involves centralized, numerical optimization, and are in this regard closely connected to recent approaches proposed by Niedringhaus [6], Durand and Alliot[8] and more recently by Frazzoli *et al.*[7].

We consider the problem of resolving conflicts arising among many aircraft following a cooperative approach, i.e. all aircraft involved in a conflict collaborate to its resolution; other cooperative approaches have been considered in ATC literature, see for example [3], [4].

The approach presented in this paper is based on the following central assumptions:

- Aircraft are assumed to cruise within a fixed altitude layer (the layer structure is the same as the one described in [4]). Aircraft can thus be modeled in a purely kinematic fashion, as points in a plane with an associated fore axis, that indicates the direction of motion, and conflict envelope radius. The task of each vehicle is to reach a given goal configuration from a given start configuration (start and goal configurations may represent waypoints planned for the aircraft by the higher level planner).
- All interacting aircraft cooperate towards optimization of a common goal, as agents in the same team. The common goal is to reach the final configuration avoiding all possible conflicts. This applies to all aircraft in the same airspace, defined as a zone in which they can exchange information on positions, velocities and goals.
- We consider two different cases: in the first case we study aircraft maneuvers consisting of instantaneous velocity changes and in the second case heading angle changes are allowed.

The problem of finding the shortest conflict-free paths, in both considered cases, can be modeled as a Mixed Integer

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Programming (MIP) problem, which may be solved using optimization tools such as CPLEX [10].

The simplicity of the model with respect to the non-linear model presented in [4], allows us to manage a large number of aircraft in the same air space. Furthermore, due to the efficient computations used to solve the problem, we can rerun the problem at regular sample times to generate a feedback control law. This leads to a straight trajectory followed with different velocities for the first case considered and to a piecewise linear trajectory in the second one.

In the case of heading angle deviation maneuvers, it is possible to add constraints in order to include forbidden sectors of the airspace in the set of non conflict constraints. Sector of airspace can be forbidden due to severe weather or crowded space. In fact such constraints are linear in the angle deviation variables. The software developed to solve both problems, written in the C language, can be easily interfaced with the FACET airspace fast-time simulation software developed at NASA [14]. This work is under way.

In [11] and [12] a similar problem was considered but in these papers the dynamic system requires fine sampling of the trajectories in order to use mixed integer programming. The approach presented in this paper does not require fine trajectory sampling. Conflict avoidance constraints for both problems considered are based on simple geometric constructions. The model based on the heading angle deviation maneuvers, in particular, could be very useful as a decision support tool for both controllers and pilot after suitable implementation studies.

This paper is organized as follows: In the second section we describe the different problems considered and the hypotheses needed to formulate them as MIP problems. In the third and fourth sections we obtain conflict avoidance constraints and formulate them as linear *or*-constraints. In section V the mixed integer programming optimization problems are provided. Numerical examples are introduced and solved using CPLEX and performance of the CPLEX resolution for different numbers of aircraft are presented in section VI for both considered problems. This section also considers the case of heading angle deviation maneuvers in which the problem of conflict avoidance is rerun every fixed time interval. After every time interval the new positions of the aircraft are considered and the new directions of flight are given by the directions of the goal configurations that they want to reach. In this case multi-segmented paths are obtained because a maneuver is allowed every fixed time interval. Furthermore the aircraft will reach the goal configuration.

II. Problem Statement

In this paper we consider a finite number n of aircraft sharing the same airspace; each aircraft is an autonomous vehicle that flies on a horizontal plane. Each aircraft has an initial and a final, desired configuration (position, heading angle) and the same goal which is to reach the final configuration in minimum time while avoiding conflicts with other aircraft. A conflict between two aircraft occurs if the aircraft are closer than a given distance d (current enroute air traffic control rules often consider this distance to be 5 nautical miles) [9].

Aircraft are identified by points in the plane (position) and angles (heading angle, direction) and thus by a point $(x, y, \theta) \in \mathbb{R} \times \mathbb{R} \times S^1$. Let $(x_i(t), y_i(t), \theta_i(t))$ be the configuration of the i-th aircraft at time t; a conflict occurs when the distance between two aircraft is less than d, i.e. a conflict between aircraft i and j occurs if for some value of t,

$$\sqrt{(x_i(t) - x_j(t))^2 + (y_i(t) - y_j(t))^2} < d. \tag{1}$$

Considering the aircraft as discs of radius $d/2$, the condition of non conflict between aircraft is equivalent to the condition of non intersection of the discs. In the following we refer to those as the *safety disc* of the aircraft. The following sections will detail the construction of linear conflict avoidance constraints that are equivalent to (1).

To avoid possible conflicts, we consider two different cases:

1. we allow aircraft to change the velocity of flight but the direction of motion remains fixed. We will refer to this case as the Velocity Change problem (VC problem);
2. aircraft fly at the same velocity v and are only allowed to change instantaneously the direction of flight. We will refer to this case as the Heading Angle Change problem (HAC problem).

In both cases each aircraft is allowed to make a maneuver, at time $t = 0$, to avoid all possible conflicts with other aircraft. We assume that no conflict occurs at time $t = 0$.

Let's define as q_i the velocity change and as p_i the heading angle deviation of the i-th aircraft. The problems consist in finding a minimum velocity change q_i (VC problem), or a minimum heading angle deviation p_i (HAC problem), for each aircraft, to avoid any possible conflict while deviating as little as possible from the original flight plan. Both problems considered can be formulated as mixed linear optimization problems with linear constraints and some boolean variables. In the following sections we, separately, formulate conflict avoidance constraints that are linear in those velocity variations q_i and angular deviations p_i.

III. Conflict avoidance constraints for the VC problem

In this section we obtain, by geometrical considerations, the conflict avoidance constraints for the VC problem. The VC problem consists of aircraft that fly along a given fixed direction and can maneuver only once with a velocity variation. The i-th aircraft changes its velocity of a quantity q_i that can be positive (acceleration), negative (deceleration) or null (no velocity variation). Each aircraft has upper and lower bounds on the velocity v_i: $v_{i,\min} \le v_i \le v_{i,\max}$. For commercial flights, during en route flight we usually have $\frac{v_{i,\max} - v_{i,\min}}{v_{i,\min}} \le 0.1$. The problem then is to find an admissible value of q_i, for each aircraft, such that all conflicts are avoided and such that the new velocity satisfies the upper and lower bounds. Hence, given the initial velocity v_i, after a velocity variation of amount q_i the following inequalities

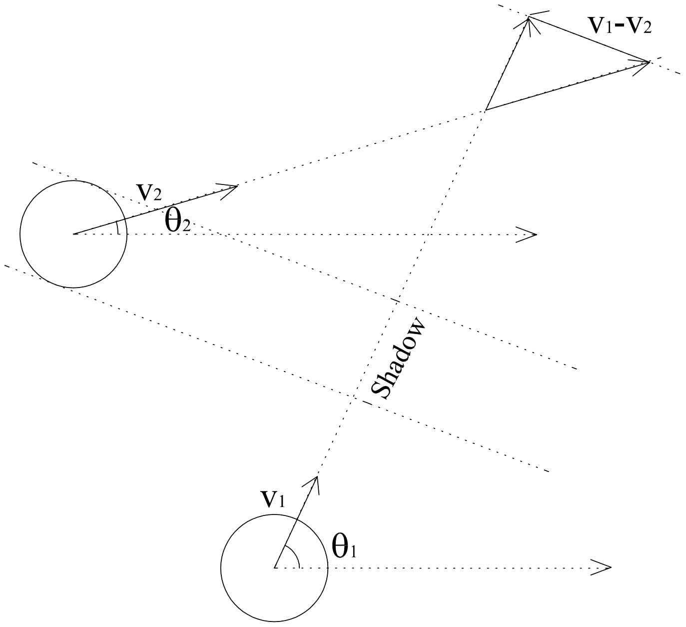

Fig. 1. Geometric construction for conflict avoidance constraints in the case of intersecting trajectories for the VC problem. In this case Aircraft 1 do not intersect the shadow generated by Aircraft 2 then no conflict will occur between the two aircraft.

must be satisfied:

$$v_{i,\min} \leq v_i + q_i \leq v_{i,\max}. \tag{2}$$

In this section we construct the conflict avoidance constraints such that are linear in the unknowns q_i, $\forall i = 1, ..., n$.

We restrict to the case of two aircraft to obtain conflict avoidance conditions and then we will consider the general case of n aircraft. Consider two aircraft denoted 1 and 2, respectively and let (x_i, y_i, θ_i), $i = 1, 2$ be the aircraft position and direction of motion and v_i be the initial velocity.

Referring to Figure 1, we consider the two velocity vectors:

$$\hat{v}_1 = \begin{pmatrix} (v_1 + q_1)\cos\theta_1 \\ (v_1 + q_1)\sin\theta_1 \end{pmatrix};$$
$$\hat{v}_2 = \begin{pmatrix} (v_2 + q_2)\cos\theta_2 \\ (v_2 + q_2)\sin\theta_2 \end{pmatrix};$$

and the difference vector:

$$\hat{v}_1 - \hat{v}_2 = \begin{pmatrix} (v_1 + q_1)\cos(\theta_1) - (v_2 + q_2)\cos(\theta_2) \\ (v_1 + q_1)\sin(\theta_1) - (v_2 + q_2)\sin(\theta_2) \end{pmatrix}.$$

The two lines parallel to $\hat{v}_1 - \hat{v}_2$ that are tangent to aircraft 2 localize a segment on the direction on motion of 1 (refer to Figure 1): we will refer to this segment as the *shadow* of aircraft 2 along the direction of 1. A conflict occurs if the aircraft 1 with his safe disc intersects the shadow generated by aircraft 2, or vice-versa since $\hat{v}_1 - \hat{v}_2$ and $\hat{v}_2 - \hat{v}_1$ are parallel.

Consider now the two non-parallel straight lines that are tangent to the discs of both aircraft (see Figure 2). Let l_{12}, r_{12} be the angles between these two straight lines and the

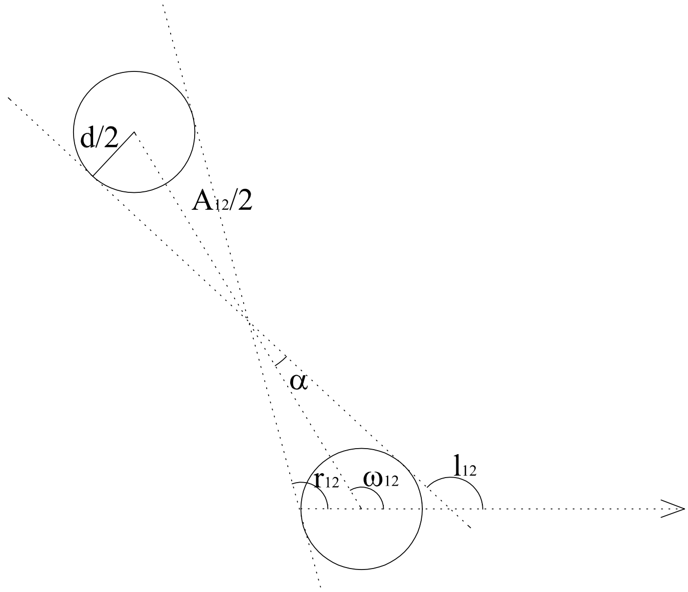

Fig. 2. The two non parallel straight lines tangent to the safety discs of radius $d/2$ for two aircraft at distance $A_{12}/2$.

horizontal axis. We have $l_{12} = \omega_{12} + \alpha$ and $r_{12} = \omega_{12} - \alpha$ with $\alpha = \arcsin\left(\frac{d}{A_{12}}\right)$ where A_{12} is the distance between the two aircraft and ω_{12} is the angle between the line that joins the aircraft and the x-axis.

No conflict occur if

$$\frac{(v_1 + q_1)\sin(\theta_1) - (v_2 + q_2)\sin(\theta_2)}{(v_1 + q_1)\cos(\theta_1) - (v_2 + q_2)\cos(\theta_2)} \geq \tan(l_{12})$$

$$\text{or} \tag{3}$$

$$\frac{(v_1 + q_1)\sin(\theta_1) - (v_2 + q_2)\sin(\theta_2)}{(v_1 + q_1)\cos(\theta_1) - (v_2 + q_2)\cos(\theta_2)} \leq \tan(r_{12})$$

To obtain non conflict constraints for n aircraft we need to consider non conflict conditions described in (3) for all pairs of aircraft. Let then consider the general pair of aircraft (i, j). We have to distinguish two possible cases: 1) $(v_i + q_i)\cos(\theta_i) - (v_j + q_j)\cos(\theta_j) < 0$ and 2) $(v_i + q_i)\cos(\theta_i) - (v_j + q_j)\cos(\theta_j) > 0$. Let $h_i = \tan(l_{ij})\cos(\theta_i) - \sin(\theta_i)$, $h_j = \tan(l_{ij})\cos(\theta_j) - \sin(\theta_j)$, $k_i = \tan(r_{ij})\cos(\theta_i) - \sin(\theta_i)$ and $k_j = \tan(r_{ij})\cos(\theta_j) - \sin(\theta_j)$, we obtain the following groups of constraints:

Case 1: $(v_i + q_i)\cos(\theta_i) - (v_j + q_j)\cos(\theta_j) < 0$

$$\begin{cases} -\cos(\theta_i)q_i + \cos(\theta_j)q_j \leq v_i\cos(\theta_i) - v_j\cos(\theta_j) \\ h_i q_i - h_j q_j \leq -v_i h_i + v_j h_j \end{cases}$$
$$\text{or} \tag{4}$$
$$\begin{cases} \cos(\theta_i)q_i - \cos(\theta_j)q_j \leq -v_i\cos(\theta_i) + v_j\cos(\theta_j) \\ -h_i q_i + h_j q_j \leq v_i h_i - v_j h_j \end{cases}$$

Case 2: $(v_i + q_i)\cos(\theta_i) - (v_j + q_j)\cos(\theta_j) > 0$

$$\begin{cases} -\cos(\theta_i)q_i + \cos(\theta_j)q_j \leq v_i\cos(\theta_i) - v_j\cos(\theta_j) \\ -q_i k_i + q_j k_j \leq v_i k_i - v_j k_j \end{cases}$$
$$\text{or} \tag{5}$$
$$\begin{cases} \cos(\theta_i)q_i - \cos(\theta_j)q_j \leq -v_i\cos(\theta_i) + v_j\cos(\theta_j) \\ q_i k_i - q_j k_j \leq -v_i k_i + v_j k_j \end{cases}$$

These two groups of constraints will be included in the model as *or*-constraints.

The case $(v_i + q_i)\cos(\theta_i) - (v_j + q_j)\cos(\theta_j) = 0$ can be easily handled, considering a rotation of the airspace such that the above condition is not satisfied. In fact the VC problem is obviously invariant with respect to rotations of the flight plane.

All constraints obtained are linear in the velocity variation q_i. To conclude the formulation of the problem we must consider the upper and lower bounds in (2) that are already linear in q_i.

If the goal of each aircraft is to avoid all possible conflicts in minimum time then we want to maximize the value of q_i such that if q_i is negative we minimize the admissible deceleration.

If we want to formulate this as a minimization problem we can chose $\sum_{i=1}^{n} -q_i$ as the cost function.

Obviously a solution to the conflict problem does not always exist, for example in the case of head-to-head conflict a change of velocity is not sufficient to avoid the conflict. Cases like the head-to-head conflicts can be easily solved with an heading angle change maneuver. In next section we then consider this kind of maneuver to avoid conflict.

IV. Conflict avoidance constraints for the HAC problem

The HAC problem consists of n aircraft that fly at the same constant velocity v and that can maneuver only once with an instantaneous heading angle deviation [7]. The i-th aircraft changes its heading angle of a quantity p_i that can be positive (left turn), negative (right turn) or null (no deviation).

The problem is then to find an admissible value of p_i for each aircraft such that all conflicts are avoided with the new heading angle (direction of flight), $\theta_i + p_i$. In this section, we formulate the non-conflict constraints for the HAC problem as inequalities that are linear in the unknowns p_i, $\forall i = 1, ..., n$ and that are function of the aircraft initial configurations $(x_i, y_i, \theta_i), i = 1, ..., n$.

As in previous section we restrict to the case of two aircraft to obtain conflict avoidance conditions and then we will consider the general case of n aircraft. Consider two aircraft denoted 1 and 2, respectively. Let $(x_i, y_i, \theta_i + p_i)$, $i = 1, 2$ be the aircraft's states after the maneuver of amplitude p_i. In this section we show that it is possible to predict the existence of conflicts between the two aircraft based on those aircraft's initial configurations. The constraints will be obtained by geometrical construction.

In order to build non conflict constraints for the HAC problem we can easily give some conditions of non conflict. In fact, considering the case of a pair of aircraft that have directions of flight that are not intersecting, we are sure that conflicts will never occur. In the following subsection we consider the case of non intersecting directions of flight and we obtain conditions of non intersections by geometrical constructions. Then the case of intersecting directions of flight will be considered.

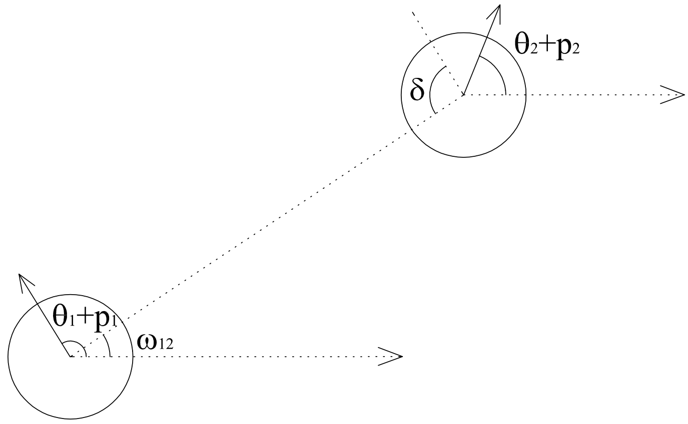

Fig. 3. Case of two aircraft, if the heading angle of Aircraft 2 does not lie in the outlined sector of amplitude δ then the trajectories do not intersect and no conflict will occur.

A. Non-intersecting directions of motion

Consider the case when the geometric half-lines representing the extrapolated trajectories of the two aircraft do not intersect. Consider for example Figure 3: aircraft 1 (on the left) has heading angle $\theta_1 + p_1$. If the heading angle $(\theta_2 + p_2)$ of the second aircraft does not lie in the outlined sector of amplitude δ then the half lines obtained by projecting forward the motion of both aircraft do not intersect. Conditions when such a case occurs can be expressed easily via some inequality constraints. Let ω_{12} be the angle between the line that joins the aircraft and the horizontal axis, two possible cases of relative positions have to be considered: case 1) $0 \le \omega_{12} \le \pi$ from the case 2) $-\pi \le \omega_{12} \le 0$. Furthermore consider the quantity $g_1 = p_1 + \theta_1 - \omega_{12}$: if $g_1 \ge \pi$ or $g_1 \le -\pi$ we have to shift the value g_1 of a quantity $-\pi$ or π respectively, so that the values that we consider lie in $[-\pi, \pi]$, while no shift for the case $-\pi \le g_1 \le \pi$ is needed. Due to those possible cases we obtain three groups of constraints for each one of the two cases of ω_{i2}. Then the following conflict avoidance conditions, linear in p_1 and p_2, are obtained by geometric construction:

Case 1: $0 \le \omega_{12} \le \pi$

$$
\begin{array}{l}
\left\{\begin{array}{l}
\omega_{12} - \pi - \theta_1 \le p_1 \le \omega_{12} - \theta_1 \\
\text{and} \\
\left\{\begin{array}{l}
\theta_1 + p_1 - \theta_2 \le p_2 \le \pi \\
\text{or} \\
-\pi - \theta_2 \le p_2 \le \omega_{12} - \pi - \theta_2;
\end{array}\right.
\end{array}\right. \\
\text{or} \\
\left\{\begin{array}{l}
\omega_{12} - \theta_1 \le p_1 \le \pi - \theta_1 \\
\text{and} \\
\omega_{12} - \pi - \theta_2 \le p_2 \le p_1 + \theta_1 - \theta_2
\end{array}\right. \\
\text{or} \\
\left\{\begin{array}{l}
-\pi - \theta_1 \le p_1 \le \omega_{12} - \pi - \theta_1 \\
\text{and} \\
\left\{\begin{array}{l}
\omega_{12} - \pi - \theta_2 \le p_2 \le \pi - \theta_2; \\
\text{or} \\
-\pi - \theta_2 \le p_2 \le p_1 + \theta_1 - \theta_2;
\end{array}\right.
\end{array}\right.
\end{array}
\tag{6}
$$

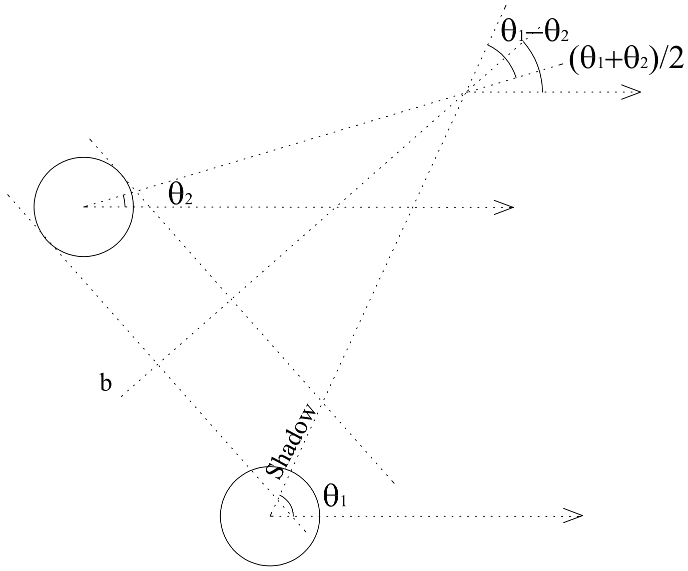

Fig. 4. Geometric construction for conflict avoidance constraints in the case of intersecting trajectories for the HAC problem. In this case the aircraft 1 intersect the shadow of aircraft 2, then a future conflict between the two aircraft has been detected.

Case 2: $-\pi \le \omega_{12} \le 0$

$$\left\{\begin{array}{l} \omega_{12}-\theta_1 \le p_1 \le \omega_{12}+\pi-\theta_1 \\ \text{and} \\ \left\{\begin{array}{l} -\pi-\theta_2 \le p_2 \le p_1+\theta_1-\theta_2 \\ \text{or} \\ \omega_{12}+\pi-\theta_2 \le p_2 \le \pi-\theta_2; \end{array}\right. \end{array}\right.$$
$$\text{or}$$
$$\left\{\begin{array}{l} -\pi-\theta_1 \le p_1 \le \omega_{12}-\theta_1 \\ \text{and} \\ p_1+\theta_1-\theta_2 \le p_2 \le \pi+\omega_{12}-\theta_2 \end{array}\right. \tag{7}$$
$$\text{or}$$
$$\left\{\begin{array}{l} \omega_{12}+\pi-\theta_1 \le p_1 \le \pi-\theta_1 \\ \text{and} \\ \left\{\begin{array}{l} -\pi-\theta_2 \le p_2 \le \omega_{12}+\pi-\theta_2; \\ \text{or} \\ p_1+\theta_1-\theta_2 \le p_2 \le \pi-\theta_2; \end{array}\right. \end{array}\right.$$

Just one of the two groups of constraints will be included in the model as *or*-constraints depending on the sign of ω_{12}. In the general case of n aircraft, we have one of those group of *or*-constraints for each pair of aircraft (i, j), for $i < j$.

B. Intersecting directions of motion

Referring to Figure 4, consider two aircraft (x_1, y_1) and (x_2, y_2) with heading angles θ_1 and θ_2 respectively. Momentarily consider $p_1 = p_2 = 0$ for simplicity (the general equation will be expressed in the next section). Consider the angle of amplitude $(\theta_1 - \theta_2)$ comprised within the aircraft flight directions. The bisector b is then a straight line that forms an angle $(\theta_1 + \theta_2)/2$ with the x-axis, while the orthogonal to the bisector forms an angle of $m_{12} = (\theta_1 + \theta_2 + \pi)/2$ with the x-axis.

The family of straight lines of slope $\tan(m_{12})$, orthogonal to the bisector, represents also the projection of one aircraft along the direction of motion of the other. The two lines in this family that are tangent to aircraft 2 localize a segment on the direction on motion of 1 (refer to Figure 4): we will refer to this segment as the *shadow* of aircraft 2 along the direction of 1. As described in section III, a conflict occurs if aircraft 1 with his safe disc intersects the shadow generated by aircraft 2, or vice-versa since the angle m_{12} is symmetric in θ_1 and θ_2.

Consider again figure 2, let $l_{12} = \omega_{12} + \alpha$ and $r_{12} = \omega_{12} - \alpha$ with $\alpha = \arcsin\left(\frac{d}{A_{12}}\right)$ where A_{12} is the distance between the two aircraft and ω_{12} is the angle between the line that joins the aircraft and the x-axis. The condition of non intersection of the shadows is equivalent to the following condition:

$$\begin{array}{l} m_{12} \le r_{12} \\ \text{or} \\ m_{12} \ge l_{12}, \end{array} \tag{8}$$

where $m_{12} = \frac{\theta_1+\theta_2+\pi}{2}$.

Consider now n aircraft and their initial configurations $(x_i, y_i, \theta_i + p_i)$, $\forall i = 1, ..., n$. We have shown in previous sections that with some geometric considerations it is possible to predict a conflict between pairs of aircraft using only information given by initial states of all n aircraft and the deviations p_i. While the constraints given by (6) and (7) are linear in the heading angle deviation p_i, the constraints obtained above are not explicity expressed in p_i. We now reformulate them as linear constraints in p_i.

Considering the general case of n aircraft and deviations p_i, from Equation (8) no conflict between the aircraft i and aircraft j occurs if

$$\begin{array}{l} \dfrac{\theta_i + p_i + \theta_j + p_j + \pi}{2} \le r_{ij} \\ \text{or} \\ \dfrac{\theta_i + p_i + \theta_j + p_j + \pi}{2} \ge l_{ij} \end{array} \tag{9}$$

where θ_i has been replaced by the new heading angle $\theta_i + p_i$ after the maneuver of amplitude p_i. Values of l_{ij} and r_{ij} are given by $\omega_{ij} \pm \arcsin\left(\frac{d}{A_{ij}}\right)$ and $l_{ij} > r_{ij}$ where $A_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$, and $\omega_{ij} = \arctan\left(\frac{y_i - y_j}{x_i - x_j}\right)$,

Let's now define $L_{ij} = l_{ij} - \frac{\theta_i+\theta_j+\pi}{2}$ and $R_{ij} = r_{ij} - \frac{\theta_i+\theta_j+\pi}{2}$. In order to avoid conflicts each pair of aircraft (i, j) with $i < j$ and such that $\frac{\theta_i+p_i+\theta_j+p_j+\pi}{2} \in [-\pi, \pi]$ must satisfy one of the following inequalities:

$$\begin{array}{l} p_i + p_j \le 2R_{ij} \\ \text{or} \\ -p_i - p_j \le -2L_{ij} \end{array} \tag{10}$$

If

$$\frac{\theta_i + p_i + \theta_j + p_j + \pi}{2} \ge \pi$$

or if

$$\frac{\theta_i + p_i + \theta_j + p_j + \pi}{2} \le -\pi$$

the quantities $R_{i,j}$ and $L_{i,j}$ must be shifted of a quantity π and $-\pi$ respectively, so that we work with angles in

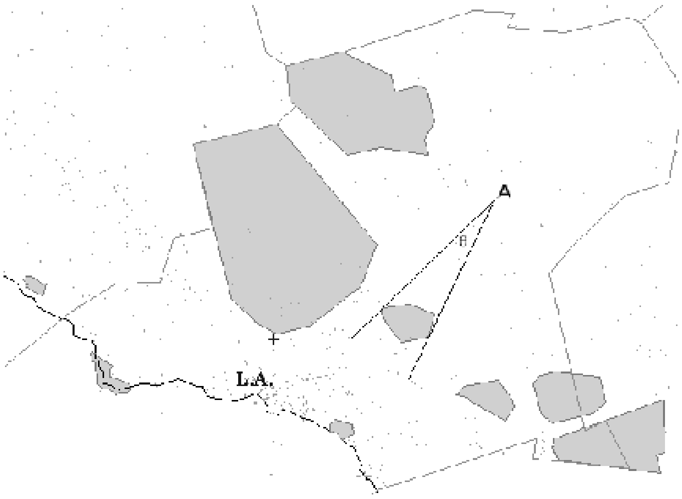

Fig. 5. Example of forbidden sectors in the Los Angeles control sector. For the aircraft A we need to introduce more constraints on the direction of flight due to forbidden zones of airspace.

$[-\pi, \pi]$. Hence considering all possible cases for the values of $\frac{\theta_i+p_i+\theta_j+p_j+\pi}{2}$ we obtain three groups of constraints:

1. Case $\frac{\theta_i+p_i+\theta_j+p_j+\pi}{2} \in [-\pi, \pi]$:

$$\begin{array}{l} p_i + p_j \leq \pi - \theta_i - \theta_j \\ -p_i - p_j \leq -3\pi + \theta_i + \theta_j \\ p_i + p_j \leq 2R_{ij} \\ \text{or} \\ p_i + p_j \leq \pi - \theta_i - \theta_j \\ -p_i - p_j \leq -3\pi + \theta_i + \theta_j \\ -p_i - p_j \leq -2L_{ij} \end{array} \tag{11}$$

2. Case $\frac{\theta_i+p_i+\theta_j+p_j+\pi}{2} > \pi$:

$$\begin{array}{l} -p_i - p_j \leq -\pi + \theta_i + \theta_j \\ p_i + p_j \leq 2R_{ij} + 2\pi \\ \text{or} \\ -p_i - p_j \leq -\pi + \theta_i + \theta_j \\ -p_i - p_j \leq -2L_{ij} - 2\pi \end{array} \tag{12}$$

3. Case $\frac{\theta_i+p_i+\theta_j+p_j+\pi}{2} < -\pi$:

$$\begin{array}{l} p_i + p_j \leq -3\pi - \theta_i - \theta_j \\ p_i + p_j \leq 2R_{ij} - 2\pi \\ \text{or} \\ p_i + p_j \leq -3\pi - \theta_i - \theta_j \\ -p_i - p_j \leq -2L_{ij} + 2\pi \end{array} \tag{13}$$

These three groups of constraints will be included in the model as *or*-constraints.

The model of the HAC problem is now complete. In the case of heading angle maneuvers we can consider in the model also other kind of constraints. For example we can consider the possible existence of forbidden zone of airspace due to sever weather or overloaded space, see Figure 5. To model those forbidden zone, it is sufficient to consider bounds of the heading angle deviations p_i.

V. Problem Formulation

The set of constraints obtained in sections III and IV are linear in the decision variables q_i and p_i for the VC and the HAC problems respectively. We now show how to recast them as mixed-integer linear constraints suitable for standard optimization software such as CPLEX [10].

A. *Writing* or*-constraints as mixed-integer programming constraints*

Let now consider, for simplpicity, an example of *or*-groups of constraints similar to the conflict avoidance constraints described in previous sections:

$$\begin{array}{l} \left\{ \begin{array}{c} c_1 \leq 0 \\ \text{and} \\ c_2 \leq 0 \end{array} \right. \\ \text{or} \\ \left\{ \begin{array}{c} c_3 \leq 0 \\ \text{and} \\ c_4 \leq 0 \\ \text{and} \\ c_5 \leq 0 \end{array} \right. \\ \text{or} \\ \left\{ \begin{array}{c} c_6 \leq 0 \\ \text{and} \\ c_7 \leq 0 \end{array} \right. \end{array} \tag{14}$$

where the terms c_i, $i = 1, \ldots, 7$ are linear expressions in the decision variables (heading angle deviations or velocity variation).

The way to transform these *or*-constraints into more convenient *and*-constraints is to introduce Boolean variables [15]. Let f_k with $k = 1, 2, 3$, be a binary number that takes value 1 when one of the *or*-constraint is active and zero otherwise (for example $f_1 = 1$ if constraints c_1 and c_2 are active, $f_1 = 0$ otherwise). Let G be a large arbitrary number, then the previous set of constraints is equivalent to:

$$\begin{array}{l} c_1 - Gf_1 \leq 0 \\ c_2 - Gf_1 \leq 0 \\ c_3 - Gf_2 \leq 0 \\ c_4 - Gf_2 \leq 0 \\ c_5 - Gf_2 \leq 0 \\ c_6 - Gf_3 \leq 0 \\ c_7 - Gf_3 \leq 0 \\ f_1 + f_2 + f_3 \leq 2 \end{array} \tag{15}$$

The last constraint indicates that at least one of the three groups of and-constraints must be verified.

B. *Variables and constraints*

Applying the procedure described in the previous section to our model of the aircraft conflict resolution and writing all the constraints in the form $a_i z_i + a_j z_j \leq b_k$ where $z_i = q_i$ for the VC problem and $z_i = p_i$ for the HAC problem, for each pair of aircraft we obtain:

- VC problem:

– 9 linear constraints, 4 from (4), 4 from (5), and 1 boolean variable constraint.
– 4 Boolean variables due to the presence of 2 groups of constraints in (4) and 2 in (5),

- HAC problem:

– 35 linear constraints, 20 from (6) or (7), 14 from (11), (12) and (13), and one constrain on the boolean variables (similar to the last one in (15)).
– 11 Boolean variables, due to the presence of 5 groups of constraints in (6) or (7), and 6 in (11), (12) and (13) that are in *or*-relation.

In section III, for the VC case, we have chosen the cost function to be $\sum_{i=1}^{n} -q_i$. Given n aircraft we have $n(n-1)/2$ aircraft pairs, resulting in a total of $n+4n(n-1)/2 = 2n^2-n$ variables and $4n(n-1)/2+2n$ constraints, the last $2n$ are due to the upper and lower bounds on the velocity.

A useful cost function for the HAC problem is to minimize the infinity norm of vector p_i, $\forall i = 1, ..., n$, i.e. minimize the $\max(|p_1|, ..., |p_n|)$. A linear cost function is obtained introducing one auxiliary variable ν such that $p_i \leq \nu$, and $-p_i \leq \nu$. Given n aircraft we have $n(n-1)/2$ aircraft pairs, resulting in a total of $n + 11n(n-1)/2 + 1$ variables and $35n(n-1)/2 + 2n$ constraints.

Another possible choice for the cost function is the 1-norm of vector $p = (p_1, ..., p_n)$, i.e. minimize $\sum_{i=1}^{n} |p_i|$; in this case n more variables must be introduced (g_i such that $p_i \leq g_i$ and $-p_i \leq g_i$, for $i = 1, ..., n$).

VI. Case studies

As shown in previous sections given the initial positions and the goal configurations of the aircraft we can easily obtain a mixed integer linear problem. In this section we report the results obtained using CPLEX to solve both the VC and the HAC problems. Unless specified, the standard value of safety distance of $5nm$ (nautical miles) has been considered. In the first group of case studies, we have considered aircraft symmetrically distributed on a circle of radius $60nm$ centered in the origin. Each aircraft is initially headed towards the origin and the goal position is the point, on the circle, that is symmetric to the initial position with respect to the origin. In the second group of case studies a general case in which no symmetry is involved and each aircraft has generic initial and final configurations has been considered. In the following, results for both problems are presented.

A. Case studies for VC problem

Consider VC problem results and the case in which all aircraft in the current configuration are on a circle centered in the origin and would have a conflict at the origin at the same time, the cases of 5 and 11 aircraft have been considered (see fig. 7 top left and top right respectively). Unpair numbers of aircraft have been chosen in order to avoid the case of head-to-head conflicts that are unsolvable in the VC case.

All aircraft are flying at velocity $8.7nm/min$ per unit of time with lower and upper bound of value 8 and $8.7nm/min$

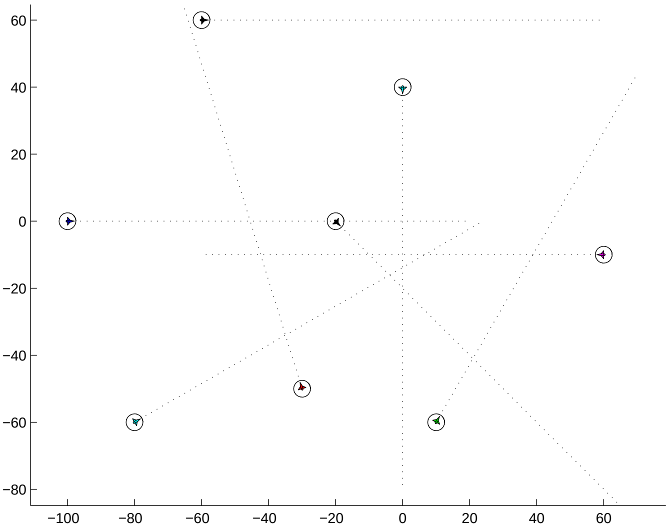

Fig. 6. Generic case of 8 aircraft in a shared airspace, three pairs of aircraft are involved in four conflicts.

. Given the cost function presented in III we want to maximize the velocity of each aircraft such that no conflict occur.

Consider also the generic case of 8 aircraft considered in figure 6 in which four pairs of aircraft are involed in three conflicts.

Results of those scenarios are reported in the following table. In the table we indicate the computational time (in seconds) of CPLEX to find the optimal solution to the MIP problem for the VC case. With # var. and # cons. we indicate the number of variables and of constraints in the VC problem respectively. Let n be the number of aircraft considered in the simulations:

n	TIME (sec)	# var.	# cons.
5	0.01	45	50
8	0.01	120	128
11	1.13	231	242

B. Case studies for HAC problem

Consider the HAC problem and the case in which, in absence of maneuvers, all aircraft would have a conflict at the origin at the same time (same scenario as the one considered for VC case studies). Figure 7 shows the symmetric scenario of the aircraft (5, 11, 13 and 15 aircraft) if no maneuver is done while in figure 8 the solutions of the HAC problem of the four scenario are plotted.

In figure 9 (top left) is shown a generic scenario of 11 aircraft in an airspace of $180nm \times 140nm$. Other aircraft have been adjoin in order to obtain a scenario with 13, 15 and 17 aircraft. In the case of 11 aircraft 4 conflict are detected, while 9, 12 and 16 conflicts are detected for the case of 13, 15 and 17 aircraft respectively. In figure 10 the solutions of the HAC problem, obtained using CPLEX, for the four scenario are plotted.

In the following table we indicate the computational time (in seconds) of CPLEX to find the optimal solution to the

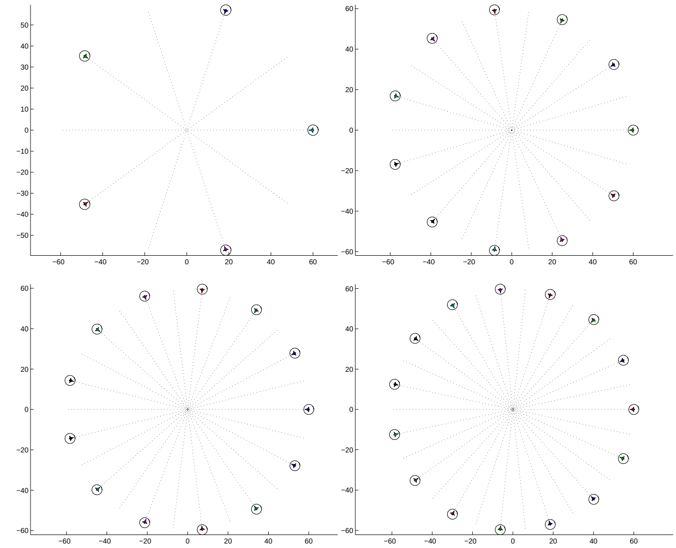

Fig. 7. We consider the case of 5,11, 13 and 15 aircraft in a symmetric configuration, the aircraft lie in a circle centered in the origin and of radius $60nm$. All the aircraft will to cross the origin at the same time.

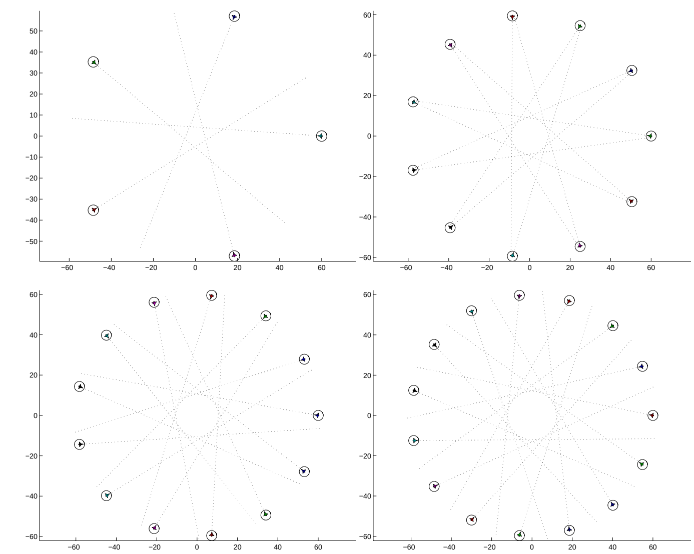

Fig. 8. In this figure is plotted the scenario of figure 7 when the aircraft has done the conflict avoidance maneuver obtained by CPLEX as the solution of the HAC problem.

MIP problem for the HAC case and the maximum angular deviation ($\Delta\theta$) from the original path. With # var. and # cons. we indicate the number of variables and of constraints in the HAC problem respectively. Let n be the number of aircraft considered in the case study; the asterisc indicates the symmetric case while 17b is the same scenario as for the 17 aircraft case (non symmetric case) in which we have considered $d = 4.5nm$ instead of $d = 5nm$. Equivalently, we have scaled the airspace such that aircraft have greater relative distances at $t = 0$. It is important to notice that the time is considerably decreased in the 17b case respect to the 17 aircraft case.

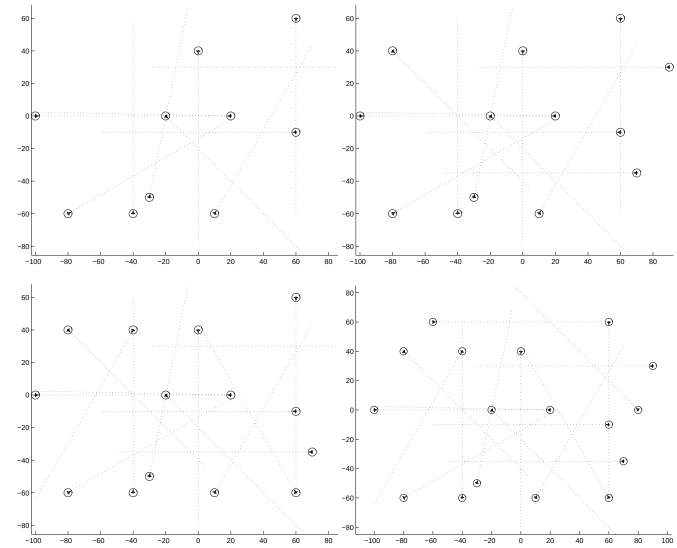

Fig. 9. We consider the case of 11, 13, 15 and 17 aircraft in a generic configuration, different conflict will occur if no conflict avoidance maneuver is done.

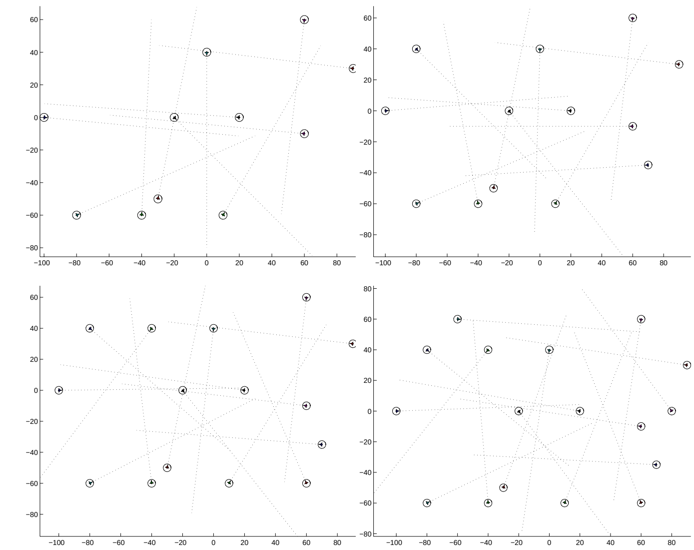

Fig. 10. In this figure is plotted the scenario of figure 9 when the aircraft have done the conflict avoidance maneuver obtained by CPLEX as the solution of the HAC problem for the different scenario.

n	TIME (sec)	$\Delta\theta$ (rad)	# var.	# cons.
5*	0.10	0.07	116	360
11	1.38	0.11	617	1947
11*	2.17	0.14	617	1947
13	4.69	0.11	872	2756
13*	8.62	0.17	872	2756
15	6.73	0.11	1171	3705
15*	15.82	0.2	1171	3705
17	13.05	0.15	1514	4794
17bis	6.82	0.14	1514	4794

The computation times are quite low, compared with other methods used to solve similar problems [7]. Thus this

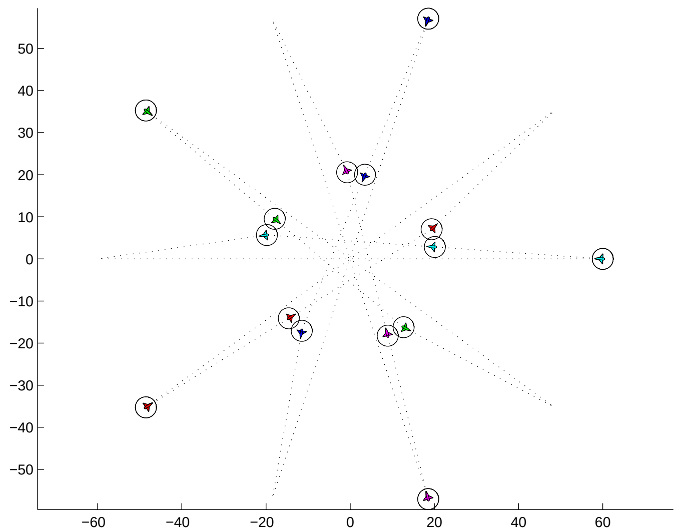

Fig. 11. Multi-segmented paths for the problem of 5 aircraft crossing the origin solved every 5 minutes, the obtained path is a multi-segmented path and the aircraft reach their final configurations.

conflict solver may be used in a real or fast-time simulation environment.

For example consider the case of 5 aircraft in the symmetric scenario plotted in figure 7 (top left). Supposing that the aircraft are flying with a velocity of $8nm/min(480nm/h)$, we solve the HAC problem every $\Delta T = 5min$ and obtain the paths illustrated in figure 11. The algorithm is rerun twice because all paths take around $15min$; in the figure we have plotted also the configurations relative to the instants in which the algorithm has been run ($t = 0$, $t = 5$, $t = 10$ minutes).

Every time the algorithm is rerun current configurations are considered and in particular the new direction of flight is the direction through the original goal configuration. In such a way aircraft will be able to reach the final destination also if have deviated from the original destination in order to solve conflicts.

The lower computational time for the VC problem respect to the HAC problem are due to the smaller number of constraints and boolean variables in the VC model as described in section V. On the contrary, the VC strategy does not solve all possible conflicts also between a pair of aircraft, as in the head-to-head conflict that is easily solved by the HAC strategy. Furthermore, also in presence of conflict between few aircraft the velocity variation strategy seems to cost more (in term of time of flight) than the heading angle deviation strategy. This is clear especially for the symmetric case considered in the simulation presented in this section. If for example we consider the case in figure 12 on the left and we solve both HAC and VC problems. In the HAC problem in order to compare the total time of flight between the two strategies, we allow aircraft to maneuver with another single heading change in order to reach the original goal configuration. This maneuver is done as soon as possible and do not generate another conflict between the two aircraft. In figure 12, on the right, the total path for the HAC case is plotted. If we consider a velocity

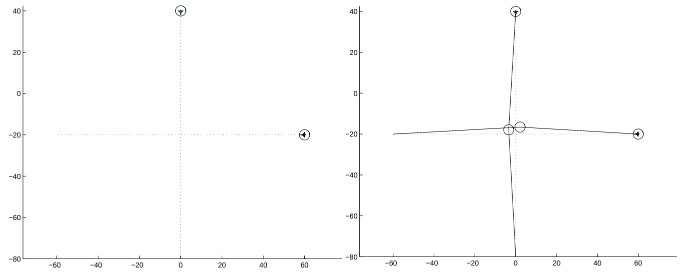

Fig. 12. On the left there is the scenario that we consider for both HAC and VC problems, on the right is the scenario for the HAC problem with the maneuver that allow aircraft to reach the final configuration.

of $8,7$nm per unit of time also in the HAC case, with the data obtained by CPLEX we can compute the total time of flight for both airplane: $13.8min$ for both airplanes in the HAC case and $13.7min$ and $15min$ for the aircraft in the VC case. Then in the HAC case we have a total flight time of $27.6min$ that is less than $28.7min$ that is the total flight time in the VC case.

VII. Conclusions and future work

Two conflict resolution maneuvers have been considered and two relative models have been presented. Based on simple geometric construction of the conflict avoidance constraints two different linear minimization problems with linear constraints and some integer variables have been obtained. The CPLEX software package has been used to solve the problem and due to the fast computation of the tool we were able to handle a large number of aircraft. Optimal solutions have been found quickly (in few seconds) on difficult cases such as the one of 15 aircraft that want to cross the same point at the same time. Future investigation of the optimal maneuver (speed change or heading angle), in term of time of flight, are part of a future work.

Due to the nonlinearity that follow from considering both heading angle and velocity variation, a future work is to consider other variables and formulate it as a mixed integer linear problem by extending the approach proposed in [6], for example.

Another issue is to consider the MIP problem as an approximation of the non linear problem presented in [4]; this can be done considering the perturbation in the right hand side vector of the MIP models.

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