Safety of a decentralized scheme for Free-Flight ATMS using Mixed Integer Linear Programming

Lucia Pallottino, Antonio Bicchi, Stefania Pancanti

Abstract

In this paper we consider policies for free-flight management of air traffic. We propose the decentralized implementation of a conflict resolution algorithm based on Mixed Integer Linear Programming techniques. The algorithm proved successful in a centralized implementation with a large number of cooperating aircraft. However, the application of such algorithm to a Free Flight environment, where cooperation can only be expected from neighboring aircraft, poses many challenges. We give here sufficient conditions under which a 3-aircraft Free Flight MILP-based scheme guarantees safety of flight.

1 Introduction

In this paper we consider multiple aircraft in a Free Flight environment where pilots are free to optimize trajectories of aircraft according to their own optimal task. Pilots are subject to the only constraint of avoiding conflicts with other aircraft. We consider a cooperative scheme in which all aircraft collaborate to solve conflicts. Conflicts are solved by pilots that take decisions autonomously based on informations available at each time. This is a decentralized ATMS scheme in which each aircraft has information of position and direction of flight of all other aircraft which are at a distance less than an "alert" radius. The problem that each single aircraft solves is based on those informations. Informations are updated every time that a new aircraft is at the alert distance from the considered aircraft. The system resulting from the above decentralized ATMS scheme is described by an hybrid system, [4]. Since safety is the major task in Free Flight we address the problem of conflict detection and solution whithin each state of the hybrid system.

In a previous work, [4], we have modeled the problem of conflict resolution in a Free Flight environment, in the case of bounded steering radius maneuvers. Necessary conditions of optimality have been found on the type of trajectories aircraft have to follow in order to avoid conflicts and minimizing the total time of flight. Safety verification of the hybrid system was very difficult to prove with the non linear model obtained. Statistical data have been furnished on safety of the decentralized scheme comparing to safety data in the centralized one. Successively, we have modeled the centralized problem in the case of instantaneous heading angle deviation maneuver, [5]. A Mixed Integer Linear Programming (MILP) problem has been obtained with a geometrical construction of non-conflict constraints.

Since the MILP model is an acceptable approximation to the bounded steering radius case, in this paper we study the decentralized scheme considering the instantaneous heading angle maneuvers. Non-conflict constraints are obtained with a geometrical construction similar to the one used in [5], although constraints obtained in this paper are more suitable for safety discussion. The problem of safe configurations within a state of the hybrid system and the problem of safe transitions from a state of the hybrid system to another is considered.

In section 2 the decentralized scheme in Free Flight is described. Geometrical construction of conflict avoidance constraints is explained in section 3 and finally in section 4 safety of the hybrid system is considered.

2 Problem Statement

We consider aircraft as autonomous vehicles that fly on an horizontal plane. Each aircraft has an initial and a final, desired configuration (position, heading angle) and the same goal: reach the final configuration in minimum time while avoiding conflicts with other aircraft. Aircraft are identified by points in the plane (position) and angles (heading angle, direction) and thus by a point $(x, y, \theta) \in \mathbb{R} \times \mathbb{R} \times S^1$. Let $(x_i(t), y_i(t), \theta_i(t))$ be the configuration of the i-th aircraft at time t; a conflict between arcraft i and j occurs if for some value of t,

$$\sqrt{(x_i(t) - x_j(t))^2 + (y_i(t) - y_j(t))^2} < d, \qquad (1)$$

where $d = 5nm$ (nautical miles), [3]. Considering the aircraft as discs of radius $d/2$, the condition of non conflict between aircraft is equivalent to the condition of non intersection of the discs. In the following we refer to those as the *safety discs* of the aircraft.

We restrict to consider aircraft that fly at the same velocity v and are allowed to change instantaneously the direction of flight to avoid possible conflicts.

In decentralized ATMS schemes, each agent (aircraft) is allowed to take decisions autonomously, based on the information that is available at each time. Several models of decentralized ATC are conceivable, which may differ in the degree of cooperative/competitive behaviour of the agents, and in the information structure ([6], [7]). In this paper, we consider a cooperative scheme which falls within the scope of the theory of teams (cf. e.g. [8], [9]). In particular, we consider the scheme presented in [4] in which:

- The i–th agent has information on the current configuration of all other agents which are at a distance less than an "alert" radius Al;
- Each agent plans its flight according to an optimal strategy which consists in minimizing the sum of the absolute values of the heading angle deviation of all aircraft the agent is aware of.

Let $S_i(\tau)$ denote the set of indices of aircraft within distance Al from the i–th aircraft at time τ. The goal of the i–th agent at time τ with information S_i is therefore to minimize

$$\mathbf{J}_{i,S_i}(\tau) = \sum_{j \in S_i} |p_j| \qquad (2)$$

subject to the non-conflict constraints.

When, during execution of flight maneuvers that were planned based on a certain information structure $I = (S_1, \ldots, S_n)$, an aircraft i with $i \notin S_j$ gets at distance Al from aircraft j, the information structure is updated, and optimal paths are replanned according to the new cost function and constraints for aircraft j.

To each different information structure I_k there corresponds a working mode for the system, i.e. dynamics driven by controls optimizing J_{i,S_i} subject to the non-conflict constraints for all pairs (i, j) with $j \in S_i$. The resulting system is composed of a finite–state machine and of associated continuous–time dynamic systems, transitions among states being triggered by conditions on the continuous variables.

For instance, in the case with $n = 3$ there are eight possible states (modes of operation), corresponding to different information structures I_k (see figure 1).

At every state transition, each agent evaluates in real–time the optimal control (heading angle change), from current information structure, for itself as well as for all other aircraft within its alert radius. Only the control policy evaluated by an agent for itself is then executed, as the one calculated for others may ignore part of the information available to them (as e.g. it happens in states I_5, I_6, and I_7 in figure 1).

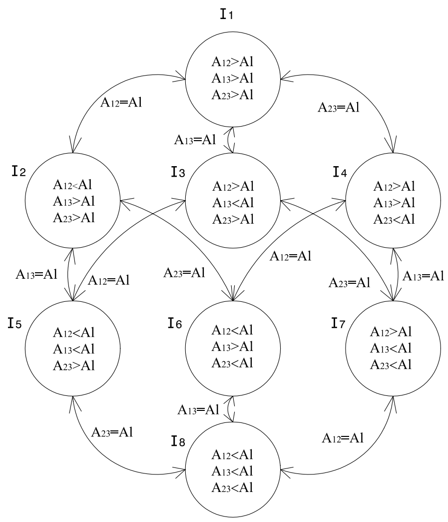

Figure 1: A decentralized ATMS with three aircraft having equal alert radius. Each node in the graph corresponds to different costs and constraints in the agents' optimal steering problem. Optimizing controllers for such problems cause different continuous time dynamics at each node. Switching between modes is triggered when an airplane enters or exits the alert neighborhood of another ($A_{ij} - Al$ changes sign).

Our goal is to study safety of the hybrid system in case of instantaneous heading angle deviation maneuvers.

3 Conflict Avoidance Constraints

In this section we introduce a geometric construction of constraints that are linear in the angular deviation of aircraft i (denoted by p_i). This geometric approach is similar to the one used in [5], but it is more suitable for safetymodified in the formalism for convenience in discussing safety of the decentralized scheme.

Let consider a general case of n aircraft that fly on a horizontal plane at the same constant velocity v and that can maneuver only once with an instantaneous heading angle deviation [10]. The i-th aircraft changes its heading angle of a quantity p_i that can be positive (left turn), negative (right turn) or null (no deviation).

The problem is then to find an admissible value of p_i for each aircraft such that all conflicts are avoided with the new heading angle (direction of flight), $\theta_i + p_i$. In this section, we formulate the non-conflict constraints as inequalities that are linear in the unknowns p_i, $\forall i = 1, ..., n$ and that are function of the aircraft initial configurations $(x_i, y_i, \theta_i), i = 1, ..., n$.

In order to construct non conflict constraints we re-

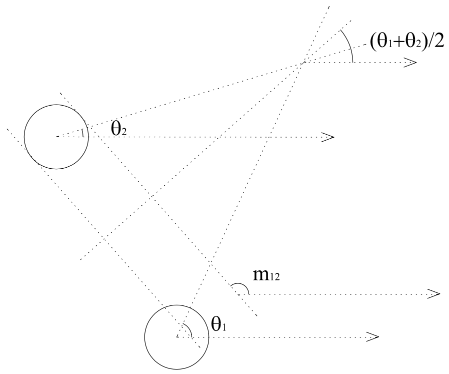

Figure 2: Geometric construction for conflict avoidance constraints. In this case the aircraft 1 intersect the shadow of aircraft 2, then a future conflict between the two aircraft has been detected.

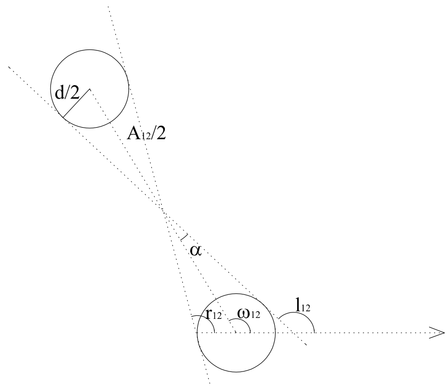

Figure 3: Geometrical construction of the two intersecting lines tangent to the safety discs of radius $d/2$ for two aircraft at distance A_{12}.

strict to the case of two aircraft. Constraints for the case of n aircraft are obtained considering all possible pairs of aircraft.

3.1 Constraints Formulation

Consider two aircraft denoted 1 and 2, respectively, for the geometric construction of constraints we refer to figure 2. Let $(x_i, y_i, \theta_i + p_i)$, $i = 1, 2$ be the aircraft's states after the maneuver of amplitude p_i. For simplicity we consider the case $p_1 = p_2 = 0$, general inequalities are easily obtainable substituiting θ_i with $\theta_i + p_i$ for $i = 1, 2$. Furthermore we consider the relative configuration of aircraft 2 respect to aircraft 1: $x_2 = x_2 - x_1$, $y_2 = y_2 - y_1$ and we consider $\theta_1 = 0$ considering $\theta_2 = \theta_2 - \theta_1$ and $\omega_{12} = \omega_{12} - \theta_1$. General constraints can be obtained with inverse transformations.

Consider coordinates $\omega_{12} = \arctan(y_2/x_2)$, $A_{12} = \sqrt{(x_2^2 + y_2^2)}$ and let the new variables of system be: ω_{12}, A_{12} and θ_2. By the geometrical construction used in [5] we have that no conflict occurs if $m_{12} = (\theta_2 + \pi)/2$ is such that

$$\begin{array}{l} m_{12} \leq r_{12} \\ \text{or} \\ m_{12} \geq l_{12}, \end{array} \tag{3}$$

where $r_{12} = \omega_{12} - \alpha_{12}$ and $l_{12} = \omega_{12} + \alpha_{12}$, with $\alpha_{12} = \arcsin\left(\frac{d}{A_{12}}\right)$ (see figures 2 and 3).

Equivalently:

$$\theta_2 \;\leq\; 2\omega_{12} - 2\alpha_{12} - \pi \tag{4}$$

or

$$\theta_2 \;\geq\; 2\omega_{12} + 2\alpha_{12} - \pi, \tag{5}$$

The geometric construction of non conflict constraints obtained above, is symmetric respect to the horizontal axis. In general, the geometric construction of non conflict constraints is symmetric respect to the straight line that forms an angle θ_1 with the horizontal axis and that pass through aircraft 1.

The symmetry, in the new variables, is equivalent to a central symmetry in the (ω_{12}, θ_2) plane, respect to the origin. Both equation (4) and (5) have been obtained by considering the case $0 \leq \omega_{12} \leq \pi$, but can be extended to the general case, $-\pi \leq \omega_{12} \leq \pi$, considering the transformation $\theta_2 = -\theta_2$ and $\omega_{12} = -\omega_{12}$:

$$\theta_2 \;\geq\; 2\omega_{12} + 2\alpha_{12} + \pi \tag{6}$$

or

$$\theta_2 \;\leq\; 2\omega_{12} - 2\alpha_{12} + \pi. \tag{7}$$

Inequalities (4), (5), (6) and (7), are represented in figure 4 for a fixed A_{12}. Outlined sectors represent configurations for which a conflict is detected. We define these configurations as *unsafe configurations.* We define *no maneuver unsafe zone* the set of configurations for which a conflict is detected while we restrict to the case $p_i = 0$. Unsafe zones in the (ω_{12}, θ_2) plane can be restricted: if $\omega_{12} \geq 0$ and $\theta_2 \geq 0$, directions of flight do not interstect hence no conflict can occur, while inequality (7) still holds. Equivalently, in the case $\omega_{12} \leq 0$, $\theta_2 \leq 0$ represents the non intersection of direction of flight condition while inequality (5) still holds. Finally, unsafe configurations are represented in figure 5.

Consider now the case $0 \leq \omega_{12} \leq \pi$. Conditions of non intersection of direction of flight and non-conflict conditions (4), (5), (6), (7) can be rewritten as groups of *and* and *or*-constraints as follow:

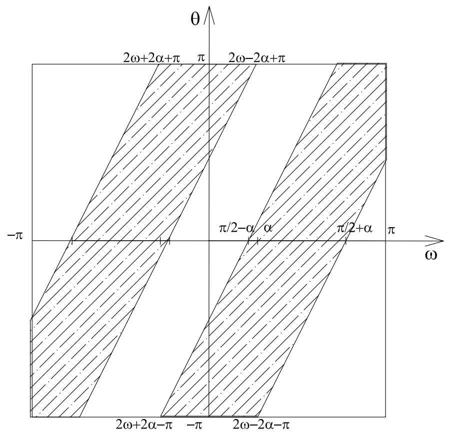

Figure 4: Representation of inequalities (4), (5), (6), (7) in the (ω, θ) plane, outline sectors represent configuration that do not verify inequalities, i.e. unsafe configurations.

$$
\begin{cases} \omega_{12} \geq \frac{\pi}{2} + \alpha_{12} \\ \text{and} \\ -\pi \leq \theta_2 \leq \pi \end{cases}
\\ \text{or} \\
\begin{cases} \omega_{12} \geq \frac{\pi}{2} - \alpha_{12} \\ \text{and} \\ \omega_{12} \leq \frac{\pi}{2} + \alpha_{12} \\ \text{and} \\ \begin{cases} 0 \leq \theta_2 \leq \pi \\ \text{or} \\ -\pi \leq \theta_2 \leq 2\omega_{12} - 2\alpha_{12} - \pi \end{cases} \end{cases}
\\ \text{or} \\
\begin{cases} \frac{\pi}{2} - \alpha_{12} \leq \omega_{12} \leq \alpha_{12} \\ \text{and} \\ 0 \leq \theta_2 \leq 2\omega_{12} - 2\alpha_{12} + \pi \end{cases}
\\ \text{or} \\
\begin{cases} \alpha_{12} \leq \omega_{12} \leq \frac{\pi}{2} - \alpha_{12} \\ \text{and} \\ \begin{cases} 2\omega_{12} + 2\alpha_{12} - \pi \leq \theta_2 \leq \pi \\ \text{or} \\ -\pi \leq \theta_2 \leq 2\omega_{12} - 2\alpha_{12} - \pi \end{cases} \end{cases}
\\ \text{or} \\
\begin{cases} 0 \leq \omega_{12} \leq \min\{\alpha_{12}, \frac{\pi}{2} - \alpha_{12}\} \\ \text{and} \\ 2\omega_{12} + 2\alpha_{12} - \pi \leq \theta_2 \leq 2\omega_{12} - 2\alpha_{12} + \pi \end{cases}
\tag{8}
$$

In the following, the set of all constraints (8) involving aircraft i and j, expressed in the original coordinates x_i, y_i, θ_i, x_j, y_j, θ_j and variables p_i, p_j, will be referred to as C_{ij}.

3.2 Conflict detection and solution

Consider the vector $P = (p_1, ..., p_n)$ of heading angle deviations for the n aircraft, it is admissible if all variables p_i verify the set of non-conflict constraints C_{ij}

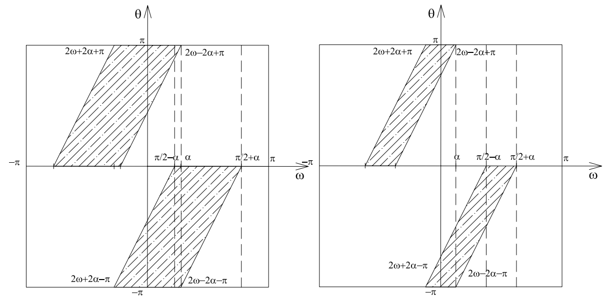

Figure 5: Given the distance A_{12} between two aircraft, unsafe configurations are given by outlined sectors. Left: case $\pi/2 - \alpha < \alpha$, Right: case $\alpha < \pi/2 - \alpha$.

for all possible pairs (i, j), otherwise it is not admissible. Consider $P_0 = (0, ..., 0)$, i.e. no aircraft deviates respect to the original direction of flight, if P_0 is admissible for the set of non-conflict constraints then no conflict has been detected, otherwise it is easy to obtain which aircraft are involved in potential conflicts. Admissible vectors P represent possible maneuvers to solve conflicts. Since usually there exists more than one admissible vector, it can be useful to look for an admissible vector that optimizes a given cost function: for example we are interested in heading angle deviations that solve conflicts and are as small as possible with respect to original directions of flight, as represented in the cost function (2). Due to the presence of *or*-constraints in the set of constraints (8), boolean variables must be introduced in order to model constraints as *and*-constraints. The model is then reformulated as a typical Mixed Integer Linear Programming (MILP) problem, in fact the precence of *or*-constraints involve the introduction on Boolean variables to model them as *and*-constraints. MILP problems can be easily solved with commercial software, due to the linear formulation of our model solutions to the conflict resolution problem can be obtained within seconds, and hence our approach is compatible with real time simulations, [5].

4 Safety of decentralized 3-aircraft system

The decentralized system in the case of three aircraft, has eight states: 1 state in which all three aircraft are at distance bigger than the alert radius (I_1), 3 cases in which two aircraft are at distance less than the alert radius while the third is at a bigger distance from the other two (I_2, I_3 and I_4), 3 cases in which one aircraft is at distance less than the alert radius from the other two while the others are at distance bigger than the alert radius from each other (I_5, I_6 and I_7), 1 case in which all the three aircraft are at distance less than the alert radius (I_8).

The problem of safe configurations within a state of the hybrid system and the problem of safe transitions from a state of the hybrid system to another is considered. In section 3, given current configurations of aircraft, we have shown how to detect potential conflicts between aircraft and we have also obtained a MILP problem. Consider configurations for which no conflict is detected, or if a conflict is detected it is solvable, i.e. a solution of the relative MILP problem exists, we will refer to this as a *safe configuration*. A transition to another state is a *safe transition* if it ends in a safe configuration of the new state of the system.

Consider the no maneuver unsafe zone given by contraints C_{ij} for the pair of aircraft (i, j). The bandwidth is given by $2\omega_{ij} + 2\alpha_{ij} - \pi - (2\omega_{ij} - 2\alpha_{ij} - \pi) = 4\alpha_{ij}$ (see figure 5), and decreases with α_{ij} or equivalently decreases as A_{ij} augments. In order to obtain unsafe zones represented in figure 5, we have considered the transformation $\theta_j = \theta_j - \theta_i$. Consider now the transformation $p_{ij} = p_j - p_i$, following the same geometrical construction described in section 3 we obtain non-conflicts constraints that are fuction of variable p_{ij}:

$$\begin{aligned} \theta_j + p_{ij} &\leq 2\omega_{ij} - 2\alpha_{ij} - \pi \\ &\text{or} \\ \theta_j + p_{ij} &\geq 2\omega_{ij} + 2\alpha_{ij} - \pi, \\ &\text{or} \\ \theta_j + p_{ij} &\geq 2\omega_{ij} + 2\alpha_{ij} + \pi, \\ &\text{or} \\ \theta_j + p_{ij} &\leq 2\omega_{ij} - 2\alpha_{ij} + \pi. \end{aligned}$$

Since $p_i \in [-p_b, p_b], \forall i$, variable $p_{ij} \in [-2p_b, 2p_b]$, then, unsafe zones in case of maneuver are determined by inequalities:

$$\begin{aligned} \theta_j &\leq 2\omega_{ij} - 2\alpha_{ij} - \pi + 2p_b \\ &\text{or} \\ \theta_j &\geq 2\omega_{ij} + 2\alpha_{ij} - \pi - 2p_b, \\ &\text{or} \\ \theta_j &\geq 2\omega_{ij} + 2\alpha_{ij} + \pi - 2p_b, \\ &\text{or} \\ \theta_j &\leq 2\omega_{ij} - 2\alpha_{ij} + \pi + 2p_b. \end{aligned}$$

and non intersrecting direction of flight conditions described in section 3. Concluding, unsafe zones in case of maneuvers are represented in outlined sectors in figure 6. For the rest of the paper, we choose $p_b = 0.35rad$ that corresponds to an istantaneous heading angle deviation of about $20deg$. The worst case occurs when the configuration of aircraft j verifies:

$$\theta_j = 2\omega_{ij} - \pi,$$

in fact, in this case maneuver $p_j = \alpha_{ij}$ or maneuver $p_j = -\alpha_{ij}$ is needed to avoid conflict, those maneuvers

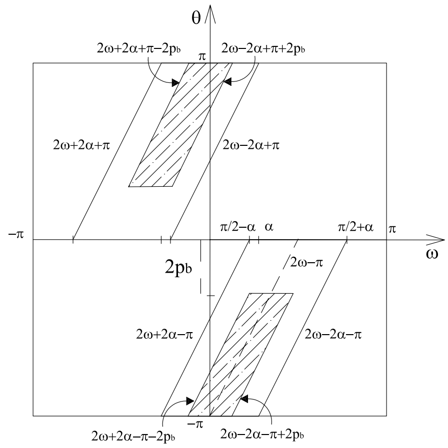

Figure 6: Unsafe zones represented in the (ω, θ) plane in the case of maxiumum heading angle deviation of amplitude $\pm 2p_b$.

are admissible if $\alpha_{ij} \leq p_b = 0.35rad$. This is true when $A_{ij} \geq 5/\sin p_b = 14.5nm$, let $d_s = 14.5nm$. Then, if two aircraft are at distance bigger than $14.5nm$, there always exists a maneuver of amplitude less than $0.35rad$ for each aircraft to avoid the conflict.

Considering the value of the alert radius to be $Al = 50nm > d_s$, if a conflict is detected between two aircraft that are at distance Al, then $\alpha = \arcsin(5nm/50nm) = 0.1rad$. If aircraft j is in a configuration that verify:

$$\theta_j = 2\omega_{ij} - 2\alpha_{ij} - \pi + 2p_b$$

then with a maneuver $p_j = +2\alpha_{ij} = 0.2rad$, as with a maneuver $p_j = -2\alpha_{ij} = -0.2rad$, the conflict is avoided. It is easy to deduce that, in any possible case, both maneuvers $0.2rad$ and $-0.2rad$ are sufficient to avoid conflicts.

Referring to figure 1, let consider now the states of the hybrid system and let determine safe transitions from state to state:

- Case I_1: in this case aircraft are at distance bigger than the alert radius, then each aircraft continue to fly taking into account only individual goals.
- Cases I_2, I_3 and I_4: in these three cases there are two aircraft that share informations while the third doesn't. Conflict between two aircraft have been solved in previous sections and in figure 6 safe and unsafe configurations are represented. Transitions from I_1 to one of those states are always safe since the transition occur if the two aircraft that share informations are at distance $Al > d_s$ then conflict is solvable with an admissible maneuver.
- Cases I_5, I_6 and I_7: a transition to one of these cases occurs if, given aircraft i, j, k, we have:

$A_{ij} < Al$, $A_{ik} > Al$ and $A_{jk} = Al$. Then, just after a transition from case I_2, I_3 or I_4, aircraft i and j have already solved their conflict, while aircraft k enters in the alert zone of j. Solution of possible conflict between j and k exists since $A_{jk} = Al$, as we have seen previously, a maneuver $p_k = \pm 0.2rad$ and $p_j = 0$ solves the conflict. If we only allow aircraft k to deviate, then no conflicts are generated between i and j since they do not maneuver, furthermore i and k are not at distance less than Al, so that they don't share any information. Hence, also in this case all transitions are safe.

- Case I_8: a transition to this state occurs if given aircraft i, j, k we have: $A_{ij} < Al$, $A_{ik} = Al$ and $A_{jk} < Al$. If configurations of i and k are such that a conflict is detected between them, we know that a maneuver of amplitude at most $0.1rad$ is sufficient to avoid conflict, it is then possible, for aircraft k, to avoid conflict with i with a maneuver of $0.2rad$ and also with a maneuver of $-0.2rad$, while $p_i = 0$. One of those choices can generate a conflict between k and j but the other one keeps j and k in a safe configuration. Transitions to this state are all safe.

5 Conclusion and future work

Conflict avoidance constraints have been obtained by a geometrical construction and are a function of current configurations and linear in the heading angle deviation variables.

The hybrid system, obtained considering the decentralized scheme in a Free Flight environment, has been considered in the case of instantaneous heading angle allowed maneuvers to solve conflicts. Based on conflict avoidance constraints obtained, safety of the hybrid system has been considered and we have demonstrated that all transitions in the hybrid system are safe.

Extensions to the general case of the hybrid system with n aircraft are under study.

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