Synergy–based Optimal Design of Hand Pose Sensing

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***Abstract*— This paper investigates the optimal design of low-cost gloves for hand pose sensing. This problem becomes particularly relevant when limits on the production costs of sensing gloves are taken into account. These cost constraints may limit both the number and the quality of sensors used as well as the technology adopted. For this reason, an optimal distribution of sensors on the glove during the design phase is mandatory in order to obtain good hand pose reconstruction. In this paper, by exploiting the knowledge on how humans most frequently use their hands in grasping tasks, we study the problem of how and where to place sensors on the glove in order to get the maximum information about the actual hand posture, and hence minimize in average the reconstruction error. Simulations and experiments of reconstruction performance are reported to validate the proposed optimal design of sensing devices.**

I. Introduction

In recent years numerous studies have underlined the complex role of human hand in motor organization, with particular attention to grasping tasks. It was shown that individuated finger motions were phylogenetically superimposed on basic grasping movements [1]. Moreover, it is possible to individuate a reduced number of coordination patterns (*synergies*) which constrain both joint motions and force exertions of multiple fingers [2]. Coordination patterns were analyzed by means of multivariate statistical methods over a grasping dataset, revealing that a limited amount of so-called *eigenpostures* or principal components (PCs) [3], [4], or otherwise *statistically identified kinematic coordination patterns* [2], are sufficient to explain a great part of hand pose variability.

In [5] we have exploited the knowledge on how humans most frequently use their hands (*a priori* information) for hand pose reconstructions from measures provided by given low–cost sensing "gloves", i.e. devices for hand pose reconstruction based on measurements of few geometric features of the hand. In this manner final results are improved in spite of insufficient and inaccurate sensing data. Glove–based systems represent the most popular devices for gesture measurement, providing useful interfaces for human–machine and haptic interaction in many fields like, for example, virtual reality, musical performance, video games, teleoperation and robotics [6]. In this paper, we extend the analysis in [5] including the optimal design of these devices. The aim is to find, for a given *a priori* information, the optimal sensor distribution that minimizes the reconstruction error in a minimum variance sense. This problem becomes particularly relevant when limits on the production costs of sensing gloves are taken into account. These cost constraints may limit both the number and the quality of sensors used, and hence an optimal distribution of sensors during the design phase is mandatory in order to achieve good performance. Notice that this optimization procedure is strictly related to the reconstruction algorithm described in [5]. Indeed, the pose reconstruction obtained using optimal sensor design is actually optimal only if reconstruction approaches described in [5] are used.

The problem of optimal design of pose sensing gloves has already been investigated, e.g. in [7], [8], [9], [10]. For instance, in [10] authors explore how to methodically select a minimal set of hand pose features from optical marker data for grasp recognition. The objective is to determine marker locations on the hand surface that is appropriate for grasp classification of hand poses. However, all the aforementioned approaches rely on experimental observations: from actual sensor data, locations that provide the largest and most useful information on the system are chosen.

In this paper we investigate a similar problem, obtaining the optimal distribution of sensors able to minimize in average the reconstruction error of hand poses, and hence maximizing the information on the real hand posture available by the glove in a minimum variance sense. Moreover, by optimizing the number and location of sensors the cost of production and the calibration time can be further reduced without loss of performance, thus enabling for device diffusion. We first consider the *continuous sensing* case, where individual sensing elements in the glove can be designed so as to measure a linear combination of joint angles. An example of this type of device is the sensorized glove developed in [11] or the 5DT Data Glove (5DT Inc., Irvine, CA, USA). However, depending on the sensing technology adopted for the glove design, the implementation of the continuous sensing can be difficult and/or costly. For this reason, we then consider the *discrete sensing* case, where each measure provided by the glove corresponds to a single joint angle. An example of this type of glove is the Humanglove (Humanware s.r.l., Pisa, Italy) or the Cyberglove (CyberGlove Systems LLC, San Jose, CA, USA). Finally, in order to take advantage from both continuous and discrete sensing — the amount of information achievable vs low-cost implementation and feasibility — we also provide an optimal design of *hybrid sensing* devices characterized by both types of sensors.

Experiments and statistical analyses demonstrate the improvement of the estimation techniques proposed in [5] by using the optimal design described in this paper.

The here discussed results can be used to enable for a more effective development of both sensorization systems

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for robotic hands and active touch sensing systems, for a wide class of applications, ranging from virtual reality to tele-robotics and rehabilitation.

II. Problem Definition

Let us consider a set of measures $y \in \mathbb{R}^m$ given by a sensing glove. By using a n Degrees of Freedom (DoFs) kinematic hand model, we assume a linear relationship between joint variables $x \in \mathbb{R}^n$ and measurements y given by

$$y = Hx + \nu\,, \tag{1}$$

where $H \in \mathbb{R}^{m\times n}$ ($m < n$) is a full row rank matrix which represents the relationship between measures and joint angles, and $\nu \in \mathbb{R}^m$ is a vector of measurement noise. Equation (1) represents a system where there are fewer equations than unknowns and hence it is compatible with an infinite number of solutions. Among these possible solutions, the least-squared solution resulting from the pseudoinverse of matrix H for system (1) is a vector of minimum Euclidean norm given by

$$\hat{x} = H^{\dagger} y\,. \tag{2}$$

However, the hand pose reconstruction obtained by (2) can be very far from the real one. In [5], among the possible solutions to (1), the most likely hand pose has been chosen to improve the reconstruction accuracy. The basic idea is to exploit the fact that human hands, although very complex and possibly different in size and shape, share many commonalities in how they are shaped and used in frequent everyday tasks. Indeed, studies on the human hand in grasping tasks have shown that multi–digit motions are strongly correlated according to some coordination patterns referred to as *postural synergies* in [4].

In [5], we have used postural synergy information embedded in the *a priori* grasp set obtained by collecting a large number N of grasp postures x_i, consisting of n DoFs, into a matrix $X \in \mathbb{R}^{n\times N}$. This information can be summarized with a covariance matrix $P_o \in \mathbb{R}^{n\times n}$, which is a symmetric matrix computed as $P_o = \frac{(X-\bar{x})(X-\bar{x})^T}{N-1}$, where $\bar{x}$ is a matrix $n \times N$ whose columns contain the mean values for each joint angle arranged in vector $\mu_o \in \mathbb{R}^n$.

Based on Minimum Variance Estimation (MVE) techniques, in [5] we have obtained the hand pose reconstruction as

$$\hat{x} = (P_o^{-1} + H^T R^{-1} H)^{-1} (H^T R^{-1} y + P_o^{-1} \mu_o)\,, \tag{3}$$

where matrix $P_p = (P_o^{-1} + H^T R^{-1} H)^{-1}$ is the *a posteriori* covariance matrix and R is the covariance matrix of noise, which is assumed to be zero–mean random Gaussian. When R tends to assume very small values, the solution described in (3) might encounter numerical problems. However, by using the Sherman-Morrison-Woodbury formulae, (3) can be rewritten as

$$\hat{x} = \mu_o - P_o H^T (H P_o H^T + R)^{-1} (H\mu_o - y)\,, \tag{4}$$

and the *a posteriori* covariance matrix becomes $P_p = P_o - P_o H^T (H P_o H^T + R)^{-1} H P_o$.

The *a posteriori* covariance matrix, which depends on measurement matrix H, represents a measure of the amount of information that an observable variable carries about unknown parameters. In this paper we investigate the role of the measurement matrix H on the estimation procedure (4), providing the optimal design of a sensing device able to obtain the maximum amount of the information on the actual hand posture.

III. Optimal Sensing Design

Let us consider the following problem to solve:

Problem 1: Let H be an $m \times n$ full row rank matrix with $m < n$ and $V_1(P_o, H, R) : \mathbb{R}^{m\times n} \to \mathbb{R}$ be defined as $V_1(P_o, H, R) = \|P_o - P_o H^T (H P_o H^T + R)^{-1} H P_o\|_F^2$, find

$$H^* = \arg\min_H V_1(P_o, H, R)$$

where $\|\cdot\|_F$ denotes the Frobenius norm defined as $\|A\|_F = \sqrt{\mathrm{tr}(AA^T)}$, for $A \in \mathbb{R}^{n\times n}$.

To solve problem 1 means to minimize the entries of the *a posteriori* covariance matrix: the smaller the values of the elements in P_p, the greater is the predictive efficiency.

To simplify the following analysis, we have separately analyzed the design of continuous, discrete and hybrid sensing devices, respectively.

A. Continuous Sensing Design

This section is dedicated to describe the closed–form solution to problem 1 for the optimal continuous sensing design, in case of both noise–free and noisy measures. In this type of glove, individual sensing elements can be designed so as to measure a linear combination of joint angles. Hereafter, we will refer to the measurement matrix related to continuous sensing devices as H_c.

Noise–Free Measures: Let A be a non-negative matrix of order n. It is well known (cf. [12]) that, for any given matrix B of rank m with $m \le n$,

$$\min_B \|A - B\|_F^2 = \alpha_{m+1}^2 + \cdots + \alpha_n^2\,, \tag{5}$$

where α_i are the eigenvalues of A, and the minimum is attained when

$$B = \alpha_1 w_1 {w_1}^T + \cdots + \alpha_m w_m {w_m}^T\,, \tag{6}$$

where w_i are the eigenvector of A associated with α_i. In other words, the choice of B as in (6) is the best fitting matrix of given rank m to A.

By using (5) and (6), the minimum of $V_1(P_o, H, 0)$ is obtained when (cf. [12])

$$\begin{aligned}\Sigma_o H^T (H \Sigma_o H^T)^{-1} H \Sigma_o = \sigma_1(P_o) u_1(P_o) u_1^T(P_o) + \cdots +\\ + \sigma_m(P_o) u_m(P_o) u_m^T(P_o)\,,\end{aligned} \tag{7}$$

where $u_i(P_o)$ is the *ith* principal components of P_o and $\sigma_i(P_o)$ its corresponding singular value. As a consequence, row vectors $(h_i)_c$ of H_c are the first m principal components of P_o, i.e. $(h_i)_c = u_i(P_o)^T$, for $i = 1, \ldots, m$.

From these results, a principal component, here refer to as synergy ([13]), can be defined as a linear combination of optimally–weighted observed variables meaning that the corresponding measures can account for a maximal amount of variance in the data set. As reported in [12], every set of

m optimal measures can be considered as a representation of points in the best fitting lower dimensional subspace. Thus the first measure gives the best one dimensional representation of data set, the first two measures give the best two dimensional representation, and so on.

Noisy Measures: Equation (6) can not be verified with noisy measures since it represents a limit case that can be achieved when H becomes very large and hence increasing the signal-to-noise ratio. To avoid this, we will present an optimal solution for problem 1 in the set $\mathscr{A} = \{H : HH^T = I_m\}$. This problem has been discussed and solved in [14], providing that, for arbitrarily noise covariance matrix R,

$$\min_{H \in \mathscr{A}} V_1(H) = \sum_{i=1}^{m} \frac{\sigma_i(P_o)}{1 + \sigma_i(P_o)/\sigma_{m-i+1}(R)} + \sum_{i=m+1}^{n} \sigma_i(P_o)\,, \quad (8)$$

and it is attained for

$$H = \sum_{i=1}^{m} u_{m-i+1}(R) u_i^T(P_o)\,. \quad (9)$$

Hence, if $\mathscr{A}$ consists of all matrices with mutually perpendicular, unit length rows, the first m principal components of P_o are always the optimal choice for H rows, even if linearly combined by the principal components of the noise covariance matrix R.

B. Discrete Sensing Design

Let us consider now the case that each measure y_j, $j = 1, \ldots, m$ provided by the glove corresponds to a single joint angle x_i, $i = 1, \ldots, n$. In this case, measurement matrix becomes a full row rank matrix where each row is a vector of the canonical basis, i.e. matrices which have exactly one nonzero entry in each row. Let H_d be a such type of matrix. The problem here is to find the optimal choice of m joints to be measured.

Let $\mathscr{O}_{m\times n}$ denotes the set of matrices with orthonormal rows, i.e. $m \times n$ matrices, with $m < n$, whose rows satisfy condition $HH^T = I_m$, and let $\mathscr{N}_{m\times n}$ denotes the set of $m \times n$ element-wise non-negative matrices; then $\mathscr{P}_{m\times n} = \mathscr{O}_{m\times n} \cap \mathscr{N}_{m\times n}$, where $\mathscr{P}_{m\times n}$ is the set of $m \times n$ matrices which have exactly one nonzero entry in each row (see [15]). In other words, if we restrict H to be orthonormal and element-wise non-negative, we obtain a matrix which has exactly one nonzero entry in each row. Hence, the problem to solve becomes:

Problem 2: Let H be a $m \times n$ matrix with $m < n$, and $V_1(P_o, H, R) : \mathbb{R}^{m\times n} \to \mathbb{R}$ be defined as $V_1(P_o, H, R) = \|P_o - P_o H^T (H P_o H^T + R)^{-1} H P_o\|_F^2$, find the optimal measurement matrix

$$H^* = \arg\min_H V_1(P_o, H, R)$$
$$s.t. \quad H \in \mathscr{P}_{m\times n}\,.$$

where $\|\cdot\|_F$ denotes the Frobenius norm defined as, $\|X\|_F = \sqrt{\mathrm{tr}(XX^T)}$, for $X \in \mathbb{R}^{n\times n}$.

The optimal measurement matrix can be computed by substituting all the possible sub–sets of m vectors of the canonical basis in $\mathbb{R}^n$ in the cost function $V_1(P_o, H, R)$, looking for the sub–set that minimizes this function, in case of both noise–free and noisy measures, respectively. More details about the optimization procedure will be included in future works.

C. Hybrid Sensing Design

According to the sensing technology of the glove, the implementation of the continuous sensing design can be difficult and/or costly, especially for a large number of measures m. For this reason, in previous section we have provided a procedure to determine which joints have to be individually measured, characterizing optimal discrete sensing devices. However, for the same number of measures m, a continuous sensing device is able to get much more information about the hand posture than a discrete one.

Therefore, in this section, for the sake of generality and in order to take advantage from both continuous and discrete sensing (the amount of information achievable vs low–cost implementation and feasibility) we consider a hybrid sensing device which combines both continuous and discrete sensors. Let us therefore define measurement matrix $H_{c,d} \in \mathbb{R}^{m\times n}$ as

$$H_{c,d} = \left[\frac{H_c}{H_d}\right],$$

where $H_c \in \mathbb{R}^{m_c\times n}$ represents the continuous sensing part and $H_d \in \mathbb{R}^{m_d\times n}$ the discrete one, with $m_c + m_d = m$.

For the hybrid sensing design, there is neither a closed–form solution nor an easy procedure to follow. Hence, taking inspiration by [15], we derive a matrix differential equation $\dot{H}_{c,d}(t) = f(P_o, R, H_{c,d})$. This equation converges to a matrix $H_{c,d}$, such that cost function $V_1(P_o, R, H_{c,d})$ is minimized, while the discrete part of the matrix also converges to the set $\mathscr{P}_{m\times n}$. We construct this matrix differential equation by combining, for appropriate cost functions, two gradient flows. The first one minimizes the cost function $V_1(P_o, H, R)$ for $H \in \mathbb{R}^{m\times n}$ as described in the following proposition.

Proposition 1: The gradient flow for the function $V_1(P_o, H, R) : \mathbb{R}^{m\times n} \to \mathbb{R}$ is given by,

$$\dot{H} = -\nabla \|P_p\|_F^2 = -4\left[P_p^2 P_o H^T \Sigma(H)\right]^T\,, \quad (10)$$

where $\Sigma(H) = (H P_o H^T + R)^{-1}$.
For sake of space we have omitted the calculation developed to obtain (10), which are based on classical matrix differentiation.

Notice that, in case of noisy measures, the minimum of $V_1(P_o, H, R)$ can not be obtained since it represents a limit case (i.e. an infimum) that can be achieved only when H becomes very large and hence increasing the signal-to-noise ratio. It is reasonable to find the optimal solution among all full row rank matrices, whose rows are unit vectors. A solution for the constrained problem can be provided by using the Rosen's gradient projection method for linear constraints [16], which is based on projecting the search direction onto the subspace tangent to the constraint itself. Furthermore, given the steepest descent direction for the unconstrained problem, this method finds the direction with the most negative directional derivative, satisfying the constraint about the structure of the H matrix. For this case, a solution can be obtained by using the projecting matrix $W = I_n - H^T(HH^T)^{-1}H$, and then projecting unconstrained gradient flow (10) onto the subspace tangent to the constraint, obtaining the search direction

$$s = \nabla \|P_p\|_F^2 W \,. \tag{11}$$

Having the search direction for the constrained problem, the gradient flow is given by

$$\dot{H} = -4 \, \left[P_p^2 P_o H^T \Sigma(H)\right]^T W \tag{12}$$

where $\Sigma(H) = (HP_oH^T + R)^{-1}$. The gradient flow (10) guarantees that the optimal solution H^* has unit vectors as rows, if $H(0)$ satisfies the latter condition.

In [15] authors define a function $V_2(P)$ with $P \in \mathbb{R}^{n\times n}$ that forces the entries of P to be as "positive" as possible. In this section, for the second gradient, we extend this function to measurement matrices $H \in \mathbb{R}^{m\times n}$ with $m < n$, thus considering function $V_2 : \mathscr{O}_{m\times n} \to \mathbb{R}$, given by

$$V_2(H) = \frac{2}{3}\,\mathrm{tr}\left[H^T(H - (H \circ H))\right] , \tag{13}$$

where $A \circ B$ denotes the *Hadamard* or elementwise product of the matrices $A = (a_{ij})$ and $B = (b_{ij})$, i.e. $A \circ B = (a_{ij}b_{ij})$. The gradient flow of $V_2(H)$ is

$$\dot{H} = -H\left[(H \circ H)^T H - H^T(H \circ H)\right] , \tag{14}$$

which minimizes $V_2(H)$ converging to a matrix in $\mathscr{P}_{m\times n}$ if $H(0) \in \mathscr{O}_{m\times n}$ ($H(0)$ is the starting point at $t = 0$ for (14)).

By combining (12) and (14), we can build the following gradient flow

$$\begin{aligned}\dot{H}_{c,d} = {} & 4\,(1-k)\, \left[P_p^2 P_o H_{c,d}^T \Sigma(H_{c,d})\right]^T W + \\ & + k\, \bar{H}_d \left[(\bar{H}_d \circ \bar{H}_d)^T \bar{H}_d - \bar{H}_d^T(\bar{H}_d \circ \bar{H}_d)\right] ,\end{aligned} \tag{15}$$

where $k \in [0,1]$ is a positive constant, $\Sigma(H_{c,d}) = (H_{c,d}P_oH_{c,d}^T + R)^{-1}$, $W = I_n - H_{c,d}^T(H_{c,d}H_{c,d}^T)^{-1}H_{c,d}$ is the projecting matrix onto the set of all matrices whose rows are unit vectors, and, finally, $\bar{H}_d$ assumed the following form:

$$\bar{H}_d = \left[\frac{0_{m_c\times n}}{H_d}\right] .$$

The gradient flow defined in (15) converges towards a hybrid sensing device, i.e. with both continuous and discrete sensors, if $H_{c,d}(0) \in \mathscr{O}_{m\times n}$, minimizing the squared Frobenius norm of the a posteriori covariance matrix. Notice that, since this problem is not convex, we can only assure that the proposed algorithm converges to a local minimum. To overcome this common problem in gradient methods a classic solution can be provided by performing a multi–start search.

IV. Results

Figure 1 shows the values of the squared norm of the *a posteriori* covariance matrix for increasing number m of measures. In particular, values of V_1 for matrices H_c^* and H_d^* are reported, for both noise–free and noisy measures. Notice that, in case of noise–free measures, V_1 values decrease with the number of measures, tending to assume nearly zero values in case of both continuous and discrete sensing. This fact is trivial because increasing the measurements we reduce the uncertainty of the measured variables. When all

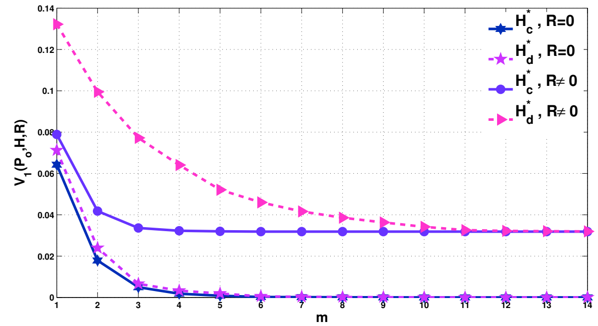


Fig. 1: Squared Frobenius norm of the *a posteriori* matrix with noise–free and noisy measures (with noise covariance matrix $R = \mathrm{diag}(12)$ [°]) for both H_c^* and H_d^*.

the measured information is available V_1 assumes zero value with perfectly accurate measures. In case of noisy measures, V_1 values decrease with the number of measures but, in this case, it tends to a value which is larger, depending on the level of noise.

In case of free-noise measures, if we analyze how much V_1 reduces with the number of measurements w.r.t. the value it assumes for only one measure, reduction percentage with three measures is greater than 80% for both H_c^* and H_d^*. This result suggests that with only three measurements the optimal matrix can furnish more than 80% of uncertainty reduction. This is equivalent to say that a reduced number of measurements is sufficient to guarantee a good hand posture estimation. In [4], [13], under the *controllability* point of view, authors state that three postural synergies are crucial in grasp pre–shaping as well as in grasping force optimization since they take into account for more than 80% of variance in grasp poses. Here, the same result can be obtained in terms of the measurement process, i.e. from the *observability* point of view: a reduced number of measures coinciding with the first three principal components enables for more than 80% reduction of the squared Frobenius norm of the *a posteriori* covariance matrix.

V. Experiments

In this work, we deal with the problem of hand pose reconstructions considering noisy measures in the discrete sensing case. We consider an additional zero–mean random Gaussian noise with standard deviation of 7° on each measure and hence a noise covariance matrix $R = \mathrm{diag}(7)$ [°]. This value is chosen in a cautionary manner, based on data about common technologies and tools used to measure hand joint positions [17]. Without loss of generality we have adopted the 15 DoF model also in used [4], [13], [5] and reported in figure 3. As described in [5], an optical motion capture system (Phase Space, San Leandro, CA - USA) with 19 active markers is used to collect a large number of static grasp positions, see figure 4. Subject AT (M,26) has performed all the grasps of the 57 imagined objects described in [4]; these data have been acquired twice to define a set of 114 *a priori* data.

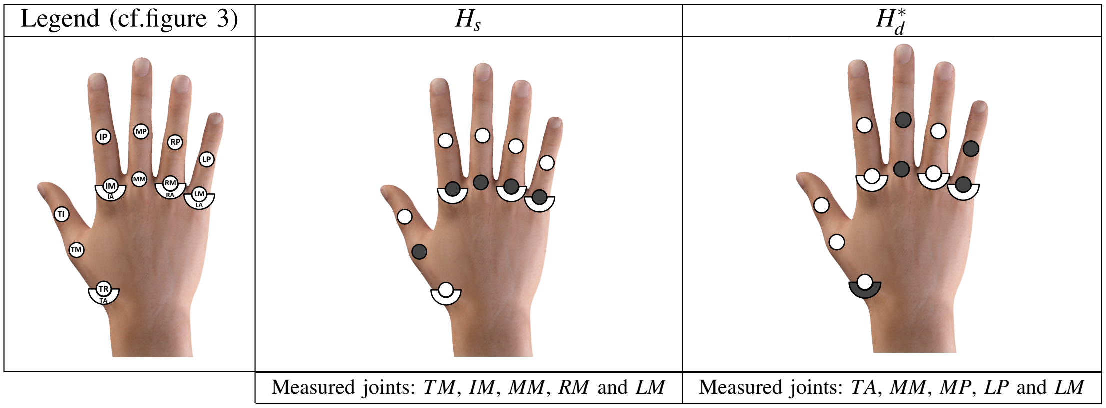
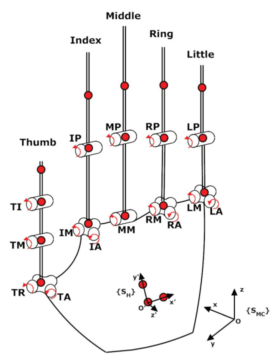


Fig. 2: Measured DoFs for matrix H_s, on the left, and H_d^*, on the right, (cf. figure 3). The measured joints are highlighted in color.



DoFs	Description
TA	Thumb Abduction
TR	Thumb Rotation
TM	Thumb Metacarpal
TI	Thumb Interphalangeal
IA	Index Abduction
IM	Index Metacarpal
IP	Index Proximal
MM	Middle Metacarpal
MP	Middle Proximal
RA	Ring Abduction
RM	Ring Metacarpal
RP	Ring Proximal
LA	Little abduction
LM	Little Metacarpal
LP	Little Proximal

Fig. 3: Kinematic model of the hand with 15 DoFs. Markers are reported as red spheres.

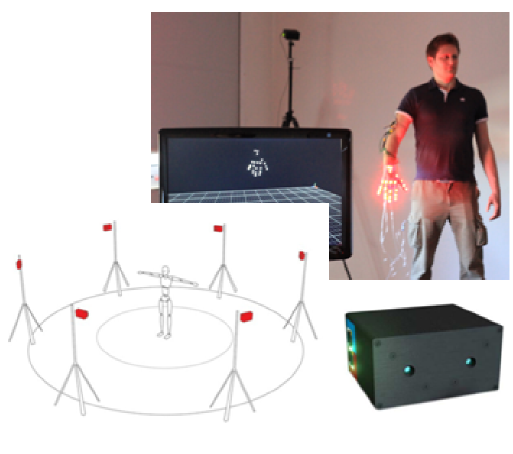


Fig. 4: Experimental setup for hand pose acquisition with Phase Space system.

The disposition of the markers on the hand refers to [18] and it is reported in figure 3. We have used four markers for the thumb and three markers for each of the rest of the fingers. Three markers have been also placed on the dorsal surface of the palm to define a local reference system S_H. The positions of the markers, which have been sampled at 480 Hz, are given referring to the global reference system S_{MC} (which is directly defined during the calibration of the acquisition system).

An additional set of $N_p = 54$ grasp poses has been performed by subject LC (26,M). Subject has been asked to perform some imagined grasped object poses contained in the *a priori* dataset and also some new postures which identify basic grasping configurations of the hand (e.g. precision and power grasp). None of the subjects had physical limitations that would affect the experimental outcomes. Data collection from subjects in this study was approved by the University of Pisa Institutional Review Board. The set of the latter poses will be referred hereinafter as *validation set*, since these poses can be assumed to represent accurate reference angular values for hand pose configurations, given the high accuracy provided by the optical system to detect markers (the amount of static marker jitter is inferior than 0.5 mm, usually 0.1 mm) and assuming a linear correlation (due to skin stretch) between marker motion around the axes of rotation of the joint and the movement of the joint itself [19]. *Validation set* has been then used to simulate discrete optimal gloves. According to the number of measures, we have considered from the postures in this set only the joints resulting from the optimization procedure, assuming to select them individually based on the non–zero elements in matrix H_d. Since all the DoFs of the postures in the *validation set* are known, we have compared the reconstructed hand configurations obtained from simulated optimal glove measures with the reference ones. Same thing has been done with hand pose reconstructions achieved starting from non optimal measures.

A. Reconstruction of Hand Postures

We have hence compared the hand posture reconstruction obtained by applying the hand pose reconstruction algorithm summarized in section II and detailed in [5] to $m = 5$ measures provided by matrix H_s, and by optimal matrix H_d^*. In figure 2 sensor locations related to matrix H_s and H_d^* are reported. H_s matrix represents the ideal case measurement matrix for the low–cost sensing glove [11] used for the experiments in [5], whereas H_d^* has been obtained by substituting all the possible sub-set of 5 vectors of the canonical basis of $\mathbb{R}^n$ in $V_1(P_o, H_d, R)$ with a noise covariance matrix $R = \mathrm{diag}(7)$ [°].

In order to compare reconstruction performance achieved with H_s and H_d^* we have used as evaluation indices, averaged over N_p, pose estimation error $e_p = \frac{1}{N_p}\sum_{k=1}^{N_p}\left(\frac{1}{n}\sum_{i=1}^{n}|x_i - \hat{x}_i|\right)$, and DoF absolute estimation error $e_d = \frac{1}{N_p}\sum_{k=1}^{N_p}|x_i - \hat{x}_i|$, for each *i–th* DoF. Maximum errors are also reported. These error indices as well as statistical tools have been chosen according to the ones considered in [5]. Statistical differences between estimated pose and joint errors obtained with the above described techniques have been computed by using classic tools, after having tested for normality and homogeneity of variances assumption on samples (through Lilliefors' composite goodness-of-fit test and Levene's test, respectively). Standard two-tailed t-test (hereinafter referred as T_{eq}) is used in case of both the assumptions are met, a modified two-tailed T-test is exploited (Behrens-Fisher problem, using Satterthwaite's approximation for the effective degrees of freedom, hereinafter referred as T_{neq}) when variance assumption is not verified and finally a non parametric test is adopted for the comparison (Mann-Whitney U-test, hereinafter referred as U) when normality hypothesis fails. Significance level of 5% is assumed and p-values less than 10^{-4} are posed equal to zero. Only noisy measures are analyzed.

In case of noise, performance in terms of pose estimation errors e_p ([°]) obtained with H_d^* is better than the one exhibited by H_s (5.96±1.42 vs. 8.18±2.70). Moreover, maximum pose error with H_d^* is the smallest (9.30° vs. 15.35° observed with H_s). Statistical difference between results from H_s and H_d^* are found (p=0.001, T_{neq}).

In table I average absolute estimation errors e_d with standard deviations are reported for each DoF. For the estimated DoFs, performance with H_d^* is always better or not statistically different from the one referred to H_s, also for not-directly measured DoFs. Maximum absolute estimation errors, considered for each DoF, with H_d^* are inferior or at most comparable to the ones obtained with H_s.

The most difficult DoFs to be estimated appear to be those related to abduction–adduction movements, such as IA and LA. A possible explanation for that can be given in terms of the *a priori* covariance matrix structure [1]. Indeed, IA and LA seem to be the less correlated with the other DoFs. Furthermore, under a technological and experimental point of view, the abduction DoFs can be the most difficult to be measured. This consideration might suggest to include in the optimization function also a cost which represents the feasibility and robustness of the measurements.

Finally, in figure 8 some reconstructed poses with MVE algorithm are reported by using both H_s and H_d^* measurement matrices. These poses are chosen because they represent some of the main topologies of postures contained in the validation set. Under a qualitative point of view, what is noticeable is that reconstructed poses are not far from the real ones for both measurement matrices, even if the pose error

[1]The *a priori* data with covariance matrix P_o and the validation set are available at http:\\handcorpus.org

DoF	Mean Error [°]		H_s vs. H_d^*	Max Error [°]	
	H_s	H_d^*	**p-values**	H_s	H_d^*
TA⊗	6.7±5.62	4.87±3.57	**0.19**	23.35	15.93
TR	7.65±5.57	7.54±5.00	**0.91** ⋄	27.46	22.73
TM∘	2.81±1.75	2.63±1.90	**0.61** ⋄	7.2	8.78
TI	6.08±4.63	5.42±4.74	**0.32**	19.6	19.10
IA	10.74±5.6	11.52±5.81	**0.32**	27.31	28.46
IM∘	4.15±3.17	6.91±5.00	0.003	11.66	21.49
IP	14.61±7.93	6.61±6.01	0	31.85	38.07
MM∘⊗	4.59±3.08	4.71±3.19	**0.77**	11.43	15.72
MP⊗	13.71±8.07	4.08±2.98	0 ‡	37.61	13.71
RA	3.12±2.37	3.28±2.45	**0.71**	9.18	9.37
RM∘	4.03±3.07	6.30±4.72	0.01 ‡	12.94	12.91
RP	16.78±11.07	6.89±3.82	0 ‡	50.66	16.34
LA	8.97±5.11	9.86±5.45	**0.38** ⋄	20.86	21.48
LM∘⊗	3.82±3.05	4.82±4.30	**0.44**	11.33	14.26
LP⊗	14.64±9.68	3.94±2.95	0	48.61	11.03

1 ⟵ 0
p-values

∘ indicates a DoF measured with H_s
⊗ indicates a DoF measured with H_d^*

TABLE I: Average estimation errors and standard deviation for each DoF [∘] for the simulated acquisition considering H_s and H_d^* both with five noisy measures. Maximum errors are also reported as well as p-values from the evaluation of DoF estimation errors between H_s and H_d^*. ⋄ indicates T_{eq} test. ‡ indicates T_{neq} test. When no symbol appears near the tabulated values, U test is used. **Bold** value indicates no statistical difference between the two methods under analysis at 5% significance level. When the difference is significative, values are reported with a 10^{-4} precision. p-values less than 10^{-4} are considered equal to zero.

$e_i = \frac{1}{n}\sum_{i=1}^{n}|x_i - \hat{x}_i|$ is smaller for the optimal case. However, it may happen that some poses are estimated in a better manner using H_s. This fact would not be surprising since MVE methods are thought to minimize error statistics rather than reconstruction errors related to individual poses [20].

VI. Conclusions and Future Works

In this paper, optimal design of sensing glove with a limited number of sensors has been proposed on the basis of the minimization of the *a posteriori* covariance matrix as it results from the estimation procedure described in [5]. Optimal solutions are described for the continuous, discrete and hybrid cases. In the continuous sensing case, optimal measures are individuated by principal components of the *a priori* covariance matrix, thus suggesting the importance of postural synergies not only for hand control in grasping tasks.

Future works will aim at investigating the impact of the *a priori* grasp dataset on the reconstruction of different pose category, such e.g. American Sign Language poses.

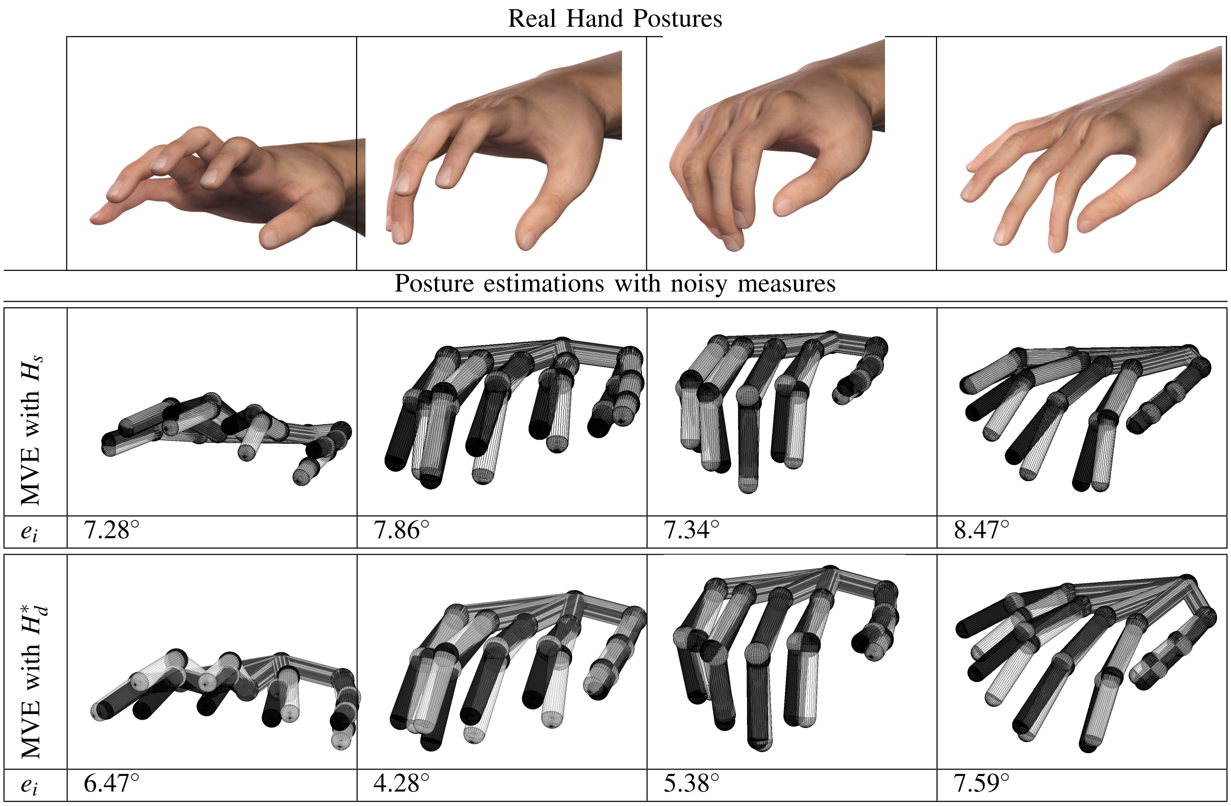


Fig. 8: Hand pose reconstructions by MVE algorithm as described in section II by using matrix H_s which allows to measure TM, IM, MM, RM and LM and matrix H_d^* which allows to measure TA, MM, MP, LP and LM (cf. figure 3). In color the real hand posture whereas in white the estimated one. The pose error is given by $e_i = \frac{1}{n}\sum_{i=1}^{n} |x_i - \hat{x}_i|$.

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