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OPTIMAL DESIGN OF DYNAMIC FORCE/TORQUE SENSORS

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Abstract: An approach to the optimal design of dynamic force/torque sensors is presented. The approach is based on the system invertibility properties of the truncated normal modes representation of an elastic beam. Theoretical results are confirmed by experiment simulations.

Keywords: Force sensors, system inversion, input estimation, optimal design.

1. INTRODUCTION

In many situations arising in robotic systems, one is interested in measuring the forces and torques applied at the terminal point of a certain mechanical structure. This information is usually obtained from strain gauges conveniently placed on the structure itself. The determination of the applied force (or torque) from the strain measurements is most often obtained by assuming a quasi-static relation between the force applied and the strain. This assumption is usually well respected even if the force is time varying, provided that the mechanical structure is rigid enough to make negligible the effects of vibration in the structure itself. This is the case, for instance, of force/torque sensors mounted at the end-effector of robot manipulators.

Whenever the flexibility of the mechanical structure can not be neglected, the measured strain will include components due to the vibration of the structure, and the determination of the applied, time-varying, force cannot be done by using quasi-static relations. This paper addresses the problem of recovering the applied force from the strain measurement by taking into account the dynamic nature of the force-strain relation, and by posing it as a problem of "system inversion" (Sain and Massey, 1969).

The determination of the applied force, whether in the quasi-static or in the full dynamic case, is dependent on the placement of the sensing device (the strain gauges) on the mechanical structure. Certain placements can be more convenient than others, in order to retrieve the force. A general setting for the optimal design of multivariate sensors is described in (Bicchi and Canepa, 1994), and its application to the specific case of quasi-static force/torque sensors is illustrated in (Bicchi, 1992). The case of optimal design of force/torque sensors in the dynamic situation is much less explored, at least in the robotic literature. The objective of this paper is to investigate the optimal design of a dynamic force/torque sensor by applying general concepts of sensor design optimization to the dynamic system relating the applied force(s) to the measured strain.

The analysis carried out in the rest of the paper specializes to the case of a flexible beam, and to the study of transversal vibrations . As such, it has to be considered as a preliminary investigation in order to proceed in the future towards more complex situations. However, even in this simple case, it appears the interesting result that the sensor design has to trade-off between the best accuracy of the solution and the stability of the inversion

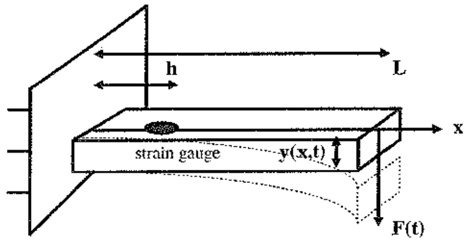

Fig. 1. The physical situation considered: a beam of length L, fixed at one end, free at the other end, is subject to a time-varying force $F(t)$ applied at the free-end that causes transverse vibrations $y(x,t)$. A strain gauge, placed at a position h along the beam, is used to estimate the applied force.

process.

The paper is organized as follows: in the next section, by using standard tools from truncated modal analysis, a system of ordinary differential equations is obtained for the case of a flexible beam. In section 3, the invertibility properties of the system are investigated, a design criterion is proposed, and constraints on the design are discussed. The optimality criterion is derived independently from the specific algorithm subsequently employed for numerical inversion. However, the effect on the design of a change in the number of modes included in the model is emphasized. In section 4, a robust numerical algorithm for system inversion is described. Simulation results obtained with this algorithm and the design choices from section 3 are reported in section 5, and finally some conclusions and description of the future work are given.

2. MODAL ANALYSIS

The transverse vibrations $y(x,t)$ excited in a flexible beam of length L fixed at one extremity by a time-varying point force $F(t)$ applied at the free end are considered. The situation is schematically depicted in Fig.1. The beam is considered isotropic, and of total mass m homogeneously distributed. A strain gauge is placed at a position h along the beam. Our design parameter is the position of the gauge h, and our objective is the retrieval of the force $F(t)$ at any time instant t from the measurement $s(h,t)$ of the strain at the position h.

The monodimensional beam subject to transverse vibration is a well studied system (see for instance (Meirovitch, 1967)) governed by the Euler-Bernoulli equations:

$$EI\frac{\partial^4 y}{\partial x^4} + 2\xi\frac{\partial^4}{\partial x^4}\frac{\partial}{\partial t}y + m\frac{\partial^2 y}{\partial t^2} = F(t)\delta(x-L) \quad (1)$$

The solution of equation (1) can be expressed in terms of its normal modes decomposition as:

$$y(x,t) = \sum_{k=1}^{\infty} q_k(t)Y_k(x) \quad (2)$$

where the terms $q_k(t)$ play the role of a weight in time and the normal modes $Y_k(x)$, $x \in [0,L]$ are defined as:

$$\begin{aligned} &Y_k(x) = \\ &(sin(\beta_k L) - sinh(\beta_k L))(sin(\beta_k x) - sinh(\beta_k x)) + \quad (3)\\ &+(cos(\beta_k L) + cosh(\beta_k L))(cos(\beta_k x) - cosh(\beta_k x)) \end{aligned}$$

with $\beta_k, k = 1,\cdots,\infty$ solutions of the equation: $cos(\beta L)cosh(\beta L) = 1$.

By truncating the modal expansion to the N-th mode, and by defining time weights and their derivative as the state vector:

$$\mathbf{x} = [q_1(t),\cdots,q_N(t),\dot{q}_1(t),\cdots,\dot{q}_N(t)]^T \quad (4)$$

after substitution in equation (1), the following system of ordinary differential equations is obtained:

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + Bu \\ y &= C\mathbf{x} \end{aligned} \quad (5)$$

where $u(t) = F(t)$ is the input force to be estimated, $y = s(t)$ is the measurement signal from the strain gauge positioned at h, and the matrices A, B and C have the following form:

$$A = \begin{bmatrix} 0 & I_N \\ -\Lambda^2 & -2\xi\Lambda \end{bmatrix} \quad (6)$$

$$B = [0\cdots0\ Y_1(L)\cdots Y_N(L)]^T \quad (7)$$

$$C = [\frac{\partial^2 Y_1(h)}{\partial x^2}\cdots\frac{\partial^2 Y_N(h)}{\partial x^2}\ 0\cdots0] \quad (8)$$

The matrix Λ is a $N \times N$ diagonal matrix whose $k-th$ diagonal term is β_k. Note that the design parameter h, i.e., the position of the strain gauge, appears in the matrix C.

3. DYNAMC INVERSION AND OPTIMAL SENSOR DESIGN

Consider system (5), where matrices A, B, and C have been previously defined for a fixed number N of modes , so that $A \in \Re^{2N\times2N}$, $B \in \Re^{2N\times1}$, $C \in \Re^{1\times2N}$. The estimation of the input signal u, given the measurement y and the knowledge of the system structure can be cast into a problem of *system inversion*. A necessary and sufficient condition for a system to be invertible has been given by Brockett and Mesarovitch (Brockett and Mesarovic, 1965). To our particular case this condition specializes as follows:

Theorem 1. System 5 is invertible iff the matrix $M \in \Re^{4N-1 \times 2N}$ is of rank $2N$

where the matrix M is defined as:

$$M = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{2N-1}B & \cdots & \cdots & CB \\ CA^{2N}B & \cdots & \cdots & CAB \\ \vdots & \ddots & \ddots & \vdots \\ CA^{4N-1}B & \cdots & \cdots & CA^{2N-1}B \end{bmatrix} \quad (9)$$

The rank test on the matrix M is *not* the most computationally efficient way to establish if a system is invertible. Several other methods have been proposed in more recent years ((Sain and Massey, 1969), (Silverman, 1969), (Moylan, 1977), (Tan and Vandewalle, 1988), to name a few). However, it is important to remark that matrix M plays the role of the "measurement matrix" of equation (1) in (Bicchi and Canepa, 1994) (see the demonstration of the above theorem as given in (Sain and Massey, 1969) for more details on this point). Matrix M gives information on the invertibility of the system, and on the properties of the inversion result, *independently from the numerical algorithm* that will be actually used. Note that, in our case, M depends (through C) on the position h of the strain gauge along the beam. The properties of M are natural candidates for the definition of the optimal design problem, i.e., to establish design criteria for the placement of the strain gauge that are generally valid.

One possible optimal design criterion is the maximization of the system inversion accuracy through the maximization of the minimum singular values of M (Bicchi and Canepa, 1994). Hence, the optimal choice h^* of the strain gauge placement takes on the form:

$$h^* = \arg \max_{h \in [0,L]} \sigma_{min}(M(h)) \quad (10)$$

Note that, for those h such that $\sigma_{min}(M(h)) = 0$, the system is not invertible.

From an algorithmic point of view, an additional requirement of an inverse system is that the inverse system must be stable. In terms of transfer functions (since we are considering a single input - single output case) this is equivalent to say that

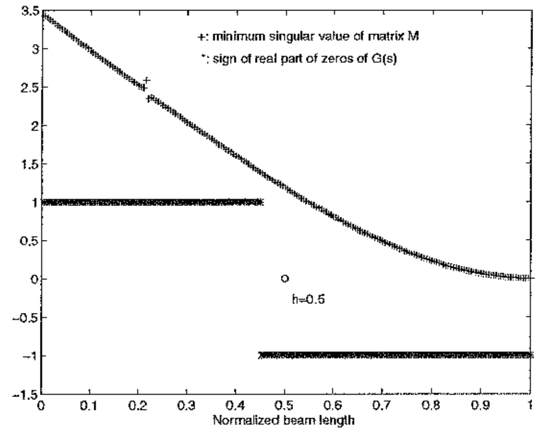

Fig. 2. Cost function and constraints for the optimal design problem, computed with a 3-modes model. +: minimum singular value of M. *: sign of the maximum real part of the zeros of $G(s)$. o: suggested sensor placement accordingly to the defined criterion

we wish that the transfer function of the system (5), for the specific choice of h selected, must *not* have any zero with positive real part. This poses an important constraint on the optimal choice of h. Having defined as $G(s) = C(sI - A)^{-1}B$ the transfer function of (5), the design criterion now becomes:

$$\begin{aligned} & h^* = \arg \max_{h \in [0,L]} \sigma_{min}(M(h)) \\ & subject\ to: \\ & Re(z(G(s))) < 0 \ \forall \ z(G(s)) \end{aligned} \quad (11)$$

The above constrained optimization problem, that in general is solved with nonlinear programming methods, can be tackled, in our simplified case, by exhaustive search and visual inspection. For instance, in Fig. 2, we report the behaviour of the minimum singular value of M as a function of the position h of the strain gauge on a normalized ($L = 1$) beam. In the same figure the sign of the maximum real part of the zeros of $G(s)$ is reported: the sign function takes on value -1 where the zeros of G(s) have all negative real part (admissible region) and $+1$ region where at least one of the zeros of $G(s)$ has positive real part (inadmissible region). From the figure, by inspection, it is clear that the maximization of the minimum singular value of M is obtained at the left extreme of the admissible region. By taking into account a normalized tolerance of 5% in the positioning of the sensor, the design choice is the placement of the strain gauge at the position $h^* = 0.5$, i.e., at the middle of the beam.

Fig. 2 has been obtained by considering the modal approximation of order 3 (i.e., $N = 3$). As the number of modes included in the approximation increases, the behaviour of the minimum singular value of M does not change. However, the posi-

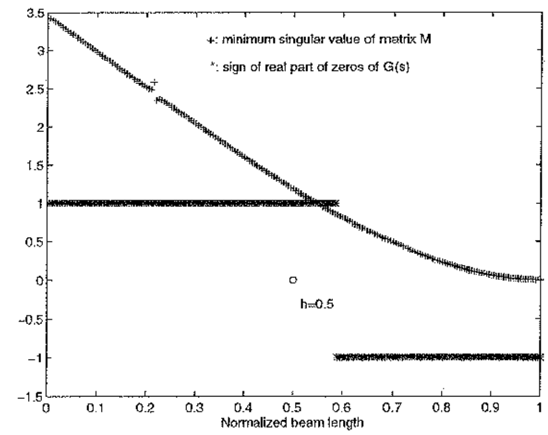

Fig. 3. Cost function and constraints for the optimal design problem computed with a 4-modes model. Symbols as in Fig. 2.

tions of the zeros of $G(s)$ in the complex plane do change: in Fig. 3 the same quantities of Fig. 2 are reported, this time obtained by considering the modal approximation of order four ($N = 4$). It can be seen that the admissible region for a stable inverse has been shrunken, and in particular the position $h^* = 0.5$ now belongs to the inadmissible region. This means that the optimal design of the sensor needs *a priori* the specification of the maximum number of modes that one wishes to invert for. Such number may be determined by the dynamic range of the strain gauge, the precision one wish to achieve, *etc.*. However, once this choice has been made, and the strain gauge placed appropriately, the sensor cannot be used to invert data by using a model with more modes than those specified in advance. Note also that, as the number of modes increases, the admissible region shrinks progressively towards the free end of the beam. In the limit case ($N \to \infty$), the only admissible point is the free end, but at the free end the system is not invertible (as seen from the minimum singular value, that approaches zero at the free end).

At this point, clearly appears that the inversion algorithm relies on a truncated modal approximation of the system. The neglected, higher order, modes will act on the inversion process as a disturbance due to *model mismatch*. One question rises naturally: how will this disturbance affect the inversion results? This will in turn depend on the numerical algorithm chosen, and imposes the requirement of a robust inversion algorithm.

4. INVERSION ALGORITHM

In order to determine the applied force, the regularized backward Euler algorithm proposed in (Caiti and Cannata, 1995) has been selected. This algorithm, originated from the numerical study of implicit (or singular) systems, has intrinsic robustness properties, and allows to estimate the input to a system with one-step delay with respect to the measured output.

From the system (5, the following implicit system is built:

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ u \end{bmatrix} + \begin{bmatrix} 0 \\ -I \end{bmatrix} y \quad (12)$$

The implicit system (12) can be written in the more compact form $E\dot{\mathbf{w}} = F\mathbf{w} + Gy$, with obvious meanings. Note that the system (12) is just a rewriting of the system (5) as an implicit system. By exploiting the non-directionality of implicit representations, it is possible to exchange the role of input and output. Moreover, the system (12) is solvable, i.e., admits unique solution, if and only if the system (5) is invertible (see for instance (Lewis *et al.*, 1987)). According to (Caiti and Cannata, 1995), the system (12) is discretized as follows:

$$\begin{aligned} &(E - \tau(\tfrac{1}{2} + \alpha)F)\mathbf{w}(j+1) = \\ &(E + \tau(\tfrac{1}{2} - \alpha)F)\mathbf{w}(j) + \\ &+\tau G[(\tfrac{1}{2} + \alpha)(y(j+1) - y(j)) + y(j)] \end{aligned} \quad (13)$$

where τ is the sampling step, and $\alpha \geq 1/2$ is the regularization parameter. For $\alpha = 1/2$ the method above is the Backward Euler method. As α increases, the solutions of the discretized equation (13) are low-pass filtered versions of the correct solution, with cut-off frequency progressively decreasing. Note also that α can be adaptively changed at each computational step.

The method (13) is simple to implement, gives the estimated input with one-step delay from the measurement $y(t)$, can be applied also in multi-input multi-output situations, and is robust with respect to disturbances thanks to the presence of the regularization parameter.

5. EXAMPLES

The following simulative cases have been considered. A beam of unitary length and homogeneous unitary mass has been considered. The damping of the system has been fixed at $\xi = 0.8$. Two time-varying forces ($F_1(t)$ and $F_2(t)$ in the following, see Fig. 4) have been selected, one continuous with discontinuous derivative, the other one discontinuous. The strain gauge has been considered as placed at $h = 0.5$, i.e., in the optimal position for a 3-mode model.

The strain gauge measurements (Fig. 5) have been generated in both cases using a truncated model of the 4-th order ($N = 4$), while the estimation has

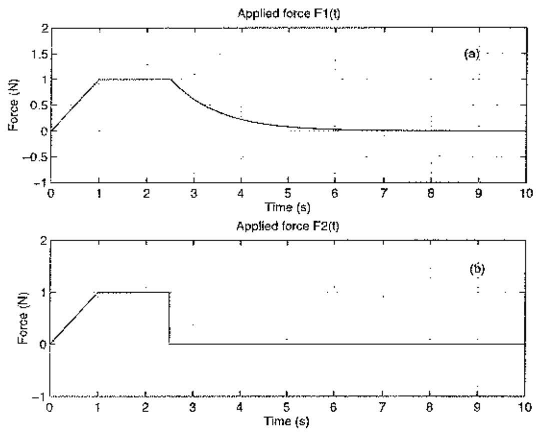

Fig. 4. (a): Force $F_1(t)$ considered in the simulative test. (b): Force $F_2(t)$ considered in the simulative test

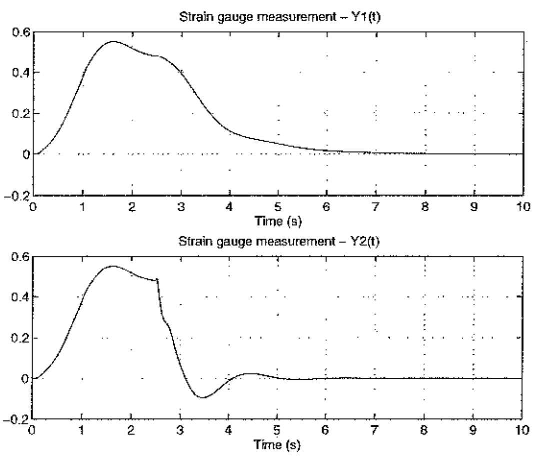

Fig. 5. (a): strain gauge measurement $y_1(t)$ obtained as a result of the application of force $F_1(t)$ using a 4 modes approximation. (b): strain gauge measurement $y_2(t)$ obtained as a result of the application of the force $F_2(t)$ using a 4 modes approximation.

been obtained by using in the inverse system (12) a 3-rd order model. This has been purposefully done in order to investigate the effect of model mismatch disturbances.

The results of the application of the algorithm (13) with $\alpha = 1/2$ (i.e., in the backward Euler case) in both cases are shown in Fig. 6. This figure has to be compared directly with Fig. 4. It can be seen that in both cases the applied forces are well reconstructed, notwithstanding the model mismatch problem, *except in the case of jump discontinuity of the applied force.* In the case of a jump discontinuity, the computed inverse solution shows, in correspondence of the jump instant, an impulse-like spurious response. However, in this case, by simply low-pass filtering the inverse solution (for instance, increasing the value of the parameter α in the algorithm (13), the signal reported in Fig. 7 is obtained. It can be seen that low pass filtering allows for a faithful reconstruction of the input force even in the case of jump discontinuity

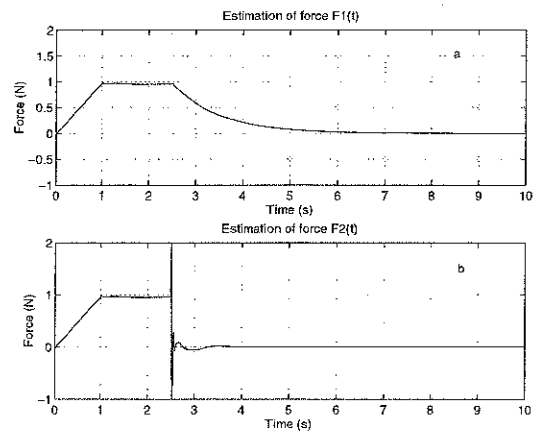

Fig. 6. (a) Reconstructed signal using a 3-rd order approximation in building the inverse model, and the backward Euler algorithm for computing the solution. The input to the algorithm is the signal (a) of Fig. 5. The desired solution is the signal (a) of Fig. 4. (b) Reconstructed signal using a 3-rd order approximation in building the inverse model, and the backward Euler algorithm for computing the solution. The input to the algorithm is the signal (b) of Fig. 5. The desired solution is the signal (b) of Fig. 4.

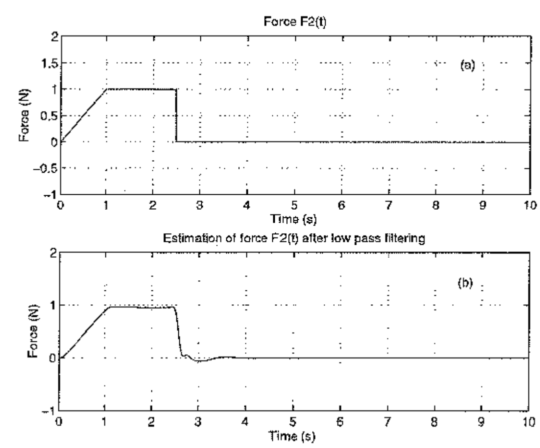

Fig. 7. (a) Desired solution ($F_2(t)$). (b) Low-pass filtered signal obtained from signal (b), Fig. 6.

6. CONCLUSIONS AND FUTURE WORK

In this paper a preliminary investigation of the problem of optimal design of dynamic force-torque sensors has been pursued. The case of a single flexible beam has been considered, and simulative results have been presented, showing that it is indeed possible to obtain robust solutions, notwithstanding the use of an approximated model in the building of the inverse system. Future developments of this work are foreseen on one side in an experimental investigation of the estimation algorithm, and on the other side, on the extension of the approach to the case of multiple force/torques acting on the system.

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