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Analysis and Control of Power Grasping

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Abstract

In this paper the problem of grasping objects with a robotic hand is considered. Unlikely most existing literature, the possibility that some or all of the fingers are not able to arbitrarily control interactions with the grasped object is taken into account. Such defective manipulation systems can still be cooperatively coordinated so as to perform usefully. In fact, having defective arms is the norm rather than an exception in many manipulation operations, such as power grasps with a hand, or whole-arm manipulation of large objects. The paper attempts to solve the problem of optimizing contact forces in the power grasp of an object. To do so, a basis of the subspace of grasp forces that are available for grasp optimisation is firstly established. This analysis is instrumental for subsequent optimization strategies, incorporating the quest for an extremum of some quality criterion. In the paper, a control algorithm is presented that guarantees the asymptotic convergence to the grasp force configuration that minimizes the risk of slippage while maintaining bounded contact forces between the fingers and the object.

1 Introduction

The problem of grasping objects with multiple coordinated robot fingers is widely recognized as fundamental in achieving higher manipulation dexterity. Since the first articulated hands appeared, a vaste literature has been produced addressing the underdetermined problem of choosing the contact forces that the fingers should apply to the grasped object in order to counterbalance given external forces acting on the object. Among the important contributions to the understanding of the problem, see e.g. [Orin and Oh, 1981], [Salisbury and Roth, 1983], [Kerr and Roth, 1986], [Li, Hsu, and Sastry, 1989], [Nakamura, Nagai, and Yoshikawa, 1989]. The common underlying idea of most of these works is that the space of contact forces can be subdivided in a subspace containing all the contact forces configurations that do not produce effects on the global balance of the grasped object. It is from this subspace that optimizing contact forces can be picked, according to the grasp quality measure that each author proposes for the different tasks at hand.

This approach assumes that any optimizing contact force can be realized by the robot fingers. For instance, if point-contacts with friction are considered, it is required that only the distal phalanges of fingers that have at least three joints in non-singular configuration are used. However, this is an overly restriction to the design and use of dextrous robot hands, since inner phalanges and also the palm can play an important role in resisting external forces and in improving the quality of grasps. A grasp that uses also such surfaces of the hand has been termed "power grasp", in view of its intrinsically better capability of bearing external loads [Cutkosky, 1989], [Mirsa and Orin, 1990].

This paper considers the grasp optimization problem taking into account also the limitations deriving from the presence of contacts on links with limited mobility, i.e. kinematically defective. Once the subspace of optimizing contact forces is thus reduced to the subspace of controllable optimizing contact forces, a grasp quality measure can be introduced that incorporates the desirable features of the grasp force configuration for the task at hand. In this paper, a control algorithm is presented that guarantees the asymptotic convergence to the grasp force configuration that minimizes the risk of slippage while maintaining bounded contact forces between the arms and the object.

2 Background

Consider an object being constrained by means of n contacts. The model of contact interactions assumed in this paper is point-contact with friction. This means that we assume that the system of forces transmitted through contact interactions is equivalent (as far as the global balance of the bodies is concerned) with a resultant force applied at the intersection of the force system's wrench axis with the body surface. Note that this contact model is not generally applicable to real contacts, for which also a local torque about the surface normal must be considered. However, the assumption is plausible for contacts between almost-rigid bodies with low friction surfaces [Bicchi, Salisbury, and Brock, 1990], and is in fact most frequently used in grasp analysis literature.

Let $\mathbf{c}_i$ be the i-th contact point, and $\mathbf{p}_i$ be the corresponding contact force (both vectors have three components). While if we are interested in an open-loop analysis of the grasp, only contact point locations need to be known (as a result of grasp planning, e.g.), whenever it is intended to realize a closed-loop control over the grasp both those locations and force components must be measured in real-time. Such information can be obtained for instance through the use of force/torque based (intrinsic) contact sensors [Bicchi, 1990], or any other equivalent device, and can be assumed to be expressed in the base reference frame.

Let the 3-vectors $\mathbf{f}$ and $\mathbf{m}$ be the net external force and moment exerted on the manipulated object, respectively. In this paper, we consider the external load as a disturbance to the firm grasp of the object, arising e.g. from the object's own weight, from inertial (D'Alembert) forces or other stimuli, to which grasp forces oppose. The balance equations for the grasped body can be written as

$$\begin{aligned} \mathbf{f} &= \sum_{i=1}^{n} \mathbf{p}_i; \\ \mathbf{m} &= \sum_{i=1}^{n} \mathbf{c}_i \times \mathbf{p}_i. \end{aligned} \tag{1}$$

or, in matrix notation, as

$$\mathbf{w} = \mathbf{G}\mathbf{t}, \tag{2}$$

where

$$\begin{aligned} \mathbf{w} &= (\mathbf{f}^T, \mathbf{m}^T)^T; \\ \mathbf{t} &= (\mathbf{p}_1^T, \mathbf{p}_2^T, \ldots, \mathbf{p}_n^T)^T; \\ \mathbf{G} &= \begin{pmatrix} \mathbf{I_3} & \mathbf{I_3} & \cdots & \mathbf{I_3} \\ \mathbf{S(c_1)} & \mathbf{S(c_2)} & \cdots & \mathbf{S(c_n)} \end{pmatrix}, \end{aligned}$$

and $\mathbf{S}(\mathbf{c_i})$ is the cross-product matrix for $\mathbf{c}_i$ (i.e. the skew-symmetric matrix such that $\mathbf{S}(\mathbf{c_i})\mathbf{v} = \mathbf{c}_i \times \mathbf{v}$). Accordingly, $\mathbf{w}$ is a 6-vector, $\mathbf{t}$ is a $3n$-vector, and $\mathbf{G}$ is a $6 \times 3n$ matrix.

Equation (2) shows that the grasp problem consists of 6 linear algebraic equations in $3n$ unknowns. For objects constrained by at least three contacts, then, the problem of finding the grasp forces $\mathbf{t}$ that can resist (or impart) a given external load $\mathbf{w}$ is generally underdetermined, and some criterion will be needed to choose among all possible solutions. In the following, we will assume that a solution to (2) exist, i.e. that $\mathbf{w} \in \mathcal{R}(\mathbf{G})$. Hence, (2) is solved in general by the sum of a particular solution $\hat{\mathbf{t}}$, and a homogeneous solution $\tilde{\mathbf{t}}$, such that

$$\mathbf{G}\hat{\mathbf{t}} = \mathbf{w}; \tag{3}$$
$$\mathbf{G}\tilde{\mathbf{t}} = 0. \tag{4}$$

The homogeneous solution corresponds to internal forces, squeezing the object in the grasp but not contributing to the overall balance. Any arrangement of contact forces resisting a given external load will be comprised of a fixed part, the particular solution $\hat{\mathbf{t}}$, and of a part that can be used for grasp optimization purposes. Note that the homogeneous solution has as many degrees of freedom as g, the nullity of matrix $\mathbf{G}$. The first condition on optimizing contact forces can therefore be written as

$$\tilde{\mathbf{t}} = \mathbf{A}\ \mathbf{x_1}, \tag{5}$$

where $\mathbf{A}$ is a $3n \times g$ matrix whose columns span the nullspace of $\mathbf{G}$, $\mathcal{N}(\mathbf{G})$, and $\mathbf{x}_1$ is a free g-vector.

Provided that enough degrees of freedom are available to arbitrarily move the finger links in contact with the object, a suitable $\tilde{\mathbf{t}}$ can be chosen to comply with other concerns about grasp forces, among which are the following:

Maximum contact forces. A delicate object could be damaged by too large grasp forces; in some cases, it is some parts of the robot system (e.g. the force sensors) that might be hurt. A safety limit, depending on the object being manipulated, should be chosen to limit the intensity of contact forces. Another reason for limiting contact forces is actuator saturation. Actuator saturation and safety bounds can be summarized as

$$\|\mathbf{p}_i\| \leq f_{i,max} > 0, i = 1, 2, \ldots, n. \tag{6}$$

where $\|\cdot\|$ indicates the euclidean norm of the argument.

Minimum contact forces. There are also reasons to keep contact forces above a minimum value. One is of practical nature: contact sensors work better in a certain range of forces, and cannot distinguish too small forces from noise. Another, perhaps deeper, reason is that we would like to avoid the *temporal discontinuity* of contacts. Klein and Kittivatcharapong [1988] designed with this term a phenomenon, consisting in a low-frequency sequence of impacts between some links of the manipulators and the object. This highly undesirable "chattering" of the contact forces has been encountered e.g. by Cheng and Orin [1989], who explained it as due to a substantial freedom in the solution of (2) while yet meeting some underconstraining optimality criterion.

A lower bound on the normal component of contact forces can be imposed as

$$\mathbf{p}_i^T \mathbf{n}_i \geq f_{i,min} > 0, \quad i = 1, 2, \ldots, n. \tag{7}$$

Friction limits. To avoid slippage at the contacts, the normal and tangential components of each contact force $\mathbf{p}_i$ must comply with Coulomb's law of friction

$$\mathbf{p}_i^T \mathbf{n}_i \geq \frac{1}{\mu_i} \|\left(\mathbf{I} - \mathbf{n}_i \mathbf{n}_i^T\right) \mathbf{p}_i\| = \alpha_i \|\mathbf{p}_i\|, \tag{8}$$

where μ_i is the static friction coefficient in the current contact conditions, and $\alpha_i = (1 + \mu_i^2)^{-1/2}$.

Task-depending Goals Depending upon the task to be pursued, different characteristics of the load distribution among the contacts are desirable. Cheng and Orin [1989], for instance, describe such optimization goals as *load balance, minimum effort*, and *temporal continuity* of contact forces. Except for the latter, this paper does not consider these and other possible goals explicitly, and focuses on the achievement of a grasp that satisfies all the constraints that are directly relevant to grasp stability. However, various other objectives could be easily added in the framework laid down in the following sections.

In summary, the constraints imposed on the choice of the grasp forces are in part expressed by linear equalities (as expressed by (2)), and partly by nonlinear inequalities, due to saturations and to friction limits. Diverse optimality criteria have been applied in literature, the basic idea being that the criterion should reflect somehow the "distance" of the grasp configuration from the constraints imposed on it. This distance can be assimilated to a grasp "stability margin", allowing for system's robustness with respect to unexpected disturbances, uncertainties of the model and sensor noise. It should be noted that often this intuitive meaning of distance from limit conditions does not reflect a precisely defined metric in the space of grasp variables. In the following, we will refer to "optimal" grasp configurations in the sense that they are "good" grasps that satisfy the constraints, and that extremize some criterion of stability.

The choice of grasp forces is a typical non-linear programming problem, and related techniques have been often applied in literature (see for instance [Jameson and Leifer,1987]; [Nakamura, Nagai, and Yoshikawa,1989]). If friction limits are linearized, the problem can be recast as a linear programming one, as for example Kerr and Roth [1986], and Cheng and Orin [1989] did.

3 Grasping with defective links

As already pointed out, most contributions to grasp force optimization so far disregarded the problem of the actual feasibility of their results on general robot hands. Thus, the results are applicable only to very special hands, i.e. those whose kinematics allow an arbitrary contact force to be realized at each contact point between the fingers and the the object. However, there are many important cases where such hypothesis does not hold, and yet the manipulation system retains its utility:

- simple grippers and medium-complexity hands, whose fingers have less then the required number of joints;
- dextrous hands using all their parts (including the inner phalanges and the palm) to achieve robust power grasps (see e.g. [Vassura and Bicchi, 1989]).

Moreover, there are other robotic applications to which the problem of "grasping" (in the broad sense) with limited mobility links is relevant, e.g.:

- cooperative manipulation with several arms, possibly comprising whole-arm manipulators ([Salisbury, 1987]);
- legged locomotion systems, using their legs and body to crawl and climb over objects;

Because of their similarity, all such robot devices can be considered to form a class of "integral" manipulation systems, to which the following analysis apply.

Consider for example the grasp of the object depicted in fig.1-a. Intuitively, there are three possible independent combinations of contact forces giving homogeneous solutions to the grasp equations, namely those lying on the edges $\mathbf{c}_1 - \mathbf{c}_2$, $\mathbf{c}_2 - \mathbf{c}_3$, $\mathbf{c}_1 - \mathbf{c}_3$ of the so-called grasp

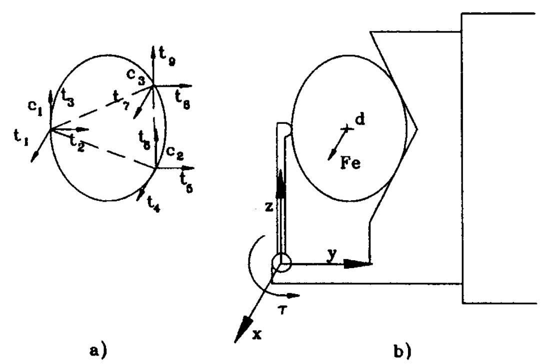

Figure 1: A simple example of grasp with limited mobility parts of a hand.

triangle. However, if the grasp is to be realized by the single-joint gripper shown in fig.1-b, it appears that some of these combinations can not be realized (e.g. opposing forces in the direction $\mathbf{c}_2 - \mathbf{c}_3$). It is also evident that the dimension of the subspace of effectively realizable internal contact forces can never be larger than the number of independent joints in the hand, i.e. in this example we can only have a one-dimensional subspace. In this section we propose an algorithm for describing a basis of this subspace.

To do so, we need to write the complete quasi-static model of the hand-object system. The relationship between joint torques and contact forces on the robot links is

$$\tau = \mathbf{D}^T \mathbf{t}, \tag{9}$$

where $\mathbf{D}$ is a $3n \times m$ matrix whose elements are functions of the robot geometric parameters and joint angles, and of the contact point locations. The role of matrix $\mathbf{D}$ is similar to that of Jacobian matrices used in the analysis of conventional manipulators, and its elements can be easily derived from kinematic considerations.

The converse relationships of (2) and (9) relating infinitesimal motions of the joints (m-vector $\delta\mathbf{q}$), of the contact points ($3n$-vector $\delta\mathbf{c} = (\delta\mathbf{c}_1^T, \delta\mathbf{c}_2^T, \cdots, \delta\mathbf{c}_n^T)^T$), and of the object in cartesian space (6-vector $\delta\mathbf{u}$), can be easily obtained through the principle of virtual work as

$$\delta\mathbf{c} = \mathbf{D}\delta\mathbf{q}; \tag{10}$$

$$\delta\mathbf{c} = \mathbf{G}^T\delta\mathbf{u}. \tag{11}$$

Finally, let the model of contact interactions between the robot links and the object-environment system be summarized by the relationship

$$\mathbf{t} = \mathbf{K}\delta\mathbf{c}, \tag{12}$$

where the $3n \times 3n$ matrix $\mathbf{K}$ is the stiffness matrix of the grasp [Salisbury and Roth, 1983]. Note that the latter equation assumes the knowledge of the elastic behaviour of the mechanical system, and includes the effects of joint servoing. A comprehensive study on the evaluation and the realization of desirable stiffness matrices with articulated hands has been presented by Cutkosky and Rao [1989].

We now ask which of the contact forces can be controlled starting from inputs at the joint level. The idea is to distinguish between *directly* and *indirectly* controllable forces:

a) suppose the object is rigidly fixed to the environment, and that the active hand joints are actuated: the resulting contact forces between the links and the grasped object are considered *directly controllable*.

b) suppose now that the object is released from the rigid attachment with the environment, and is subject to the external force/torque that directly controllable contact forces would cause. Indirectly controllable contact forces are those that the mechanism would oppose to counterbalance such external force/torque.

Using (12) and (10), the subspace of directly controllable forces result to be the range space of $\mathbf{KD}$, i.e. $\mathcal{R}(\mathbf{KD})$. Therefore, a generic directly controllable contact force can be written as

$$\mathbf{t}_{dc} = \mathbf{KD}\mathbf{x}_2, \tag{13}$$

where $\mathbf{x}_2$ is a free m-vector.

The subspace of indirectly controllable contact forces $\mathbf{t}_{ic}$ can be obtained considering (12) and (11), as $\mathcal{R}(\mathbf{KG}^T)$, and hence

$$\mathbf{t}_{ic} = \mathbf{KG}^T\mathbf{x}_3, \tag{14}$$

where $\mathbf{x}_3$ is again a free 6-vector.

The subspace $\mathcal{F}$ of all realizable internal contact forces is given by all the combinations of directly and indirectly controllable forces, that are also internal; in set notation, therefore, $\mathcal{F} = \mathcal{R}(\mathbf{A}) \cap \{\mathcal{R}(\mathbf{D}) \oplus \mathcal{R}(\mathbf{G}^T)\}$. In order to find a basis of this subspace, let $\mathbf{w}'$ be the external force/torque caused by the generic directly controllable contact force :

$$\mathbf{w}' = \mathbf{G}\mathbf{t}_{dc}.$$

Indirectly controllable contact forces excited by $\mathbf{w}'$ must be mapped under $\mathbf{G}$ in the same $\mathbf{w}'$, i.e., using (13) and (14),

$$\mathbf{GKD}\mathbf{x}_2 = \mathbf{w}' = \mathbf{GKG}^T\mathbf{x}_3. \tag{15}$$

or, subtracting the left hand side from the first, and factoring the matrix $\mathbf{G}$ out,

$$\mathbf{G}(\mathbf{KD}\mathbf{x}_2 - \mathbf{KG}^T\mathbf{x}_3) = 0. \tag{16}$$

According to the above definition of the matrix $\mathbf{A}$, (16) can be rewritten as

$$\mathbf{A}\mathbf{x}_1 = \mathbf{KD}\mathbf{x}_2 - \mathbf{KG}^T\mathbf{x}_3,$$

In block matrix form we write this equation as

$$\begin{bmatrix} \mathbf{A} & -\mathbf{KD} & \mathbf{KG}^T \end{bmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{pmatrix} = 0. \tag{17}$$

Let $\mathbf{Q} = \begin{bmatrix} \mathbf{A} & -\mathbf{KD} & \mathbf{KG}^T \end{bmatrix}$ ($\mathbf{Q}$ is a $3n \times (g+m+6)$ matrix), and $\mathbf{B}$ a $(g+m+6) \times q$ matrix whose columns span the nullspace of $\mathbf{Q}$ (whose nullity is q). Finally, partition $\mathbf{B}$ as $\mathbf{B} = \begin{bmatrix} \mathbf{B}_1^T & \mathbf{B}_2^T & \mathbf{B}_3^T \end{bmatrix}^T$, where $\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{B}_3$, are respectively $g \times q$, $m \times q$, and $6 \times q$ blocks. All controllable homogeneous (internal) forces can therefore be expressed as

$$\tilde{\mathbf{t}}_r = \mathbf{E}\mathbf{y}, \tag{18}$$

where the columns of the $3n \times h$ matrix $\mathbf{E}$ form a basis of the range of $\mathbf{AB}_1$, i.e. of $\mathcal{F}$, and $\mathbf{y}$ is a free h-vector ($h \le q$).

Note that the vector $\mathbf{y}$ is comprised of the h free variables that can be used to set up an unconstrained grasp optimization problem, as it will be described in the following section. In fact, since a particular solution $\hat{\mathbf{t}}$ to the object balance equation (2) is assumed to exist, all realizable solutions to the object balance equation (2) can be written as

$$\mathbf{t}_r = \hat{\mathbf{t}} + \tilde{\mathbf{t}}_r = \mathbf{G}^L\mathbf{w} + \mathbf{E}\mathbf{y}, \tag{19}$$

where $\mathbf{G}^L$ indicates a generalized left inverse of the grasp matrix $\mathbf{G}$.

Example. To illustrate the above relationships, consider again the example of fig.1. Let $\mathbf{c}_1 = (0\ 0\ 2a)^T$; $\mathbf{c}_2 = (0\ 2a\ a)^T$; $\mathbf{c}_3 =$

$(0\ 2a\ 3a)^T$. The corresponding unit normal vectors are $\mathbf{n}_1 = (010)^T$; $\mathbf{n}_2 = (0\ -\frac{\sqrt{3}}{2}\ \frac{1}{2})^T$; $\mathbf{n}_3 = (0\ -\frac{\sqrt{3}}{2}\ -\frac{1}{2})^T$. Finally, assume that the grasp stiffness matrix is diagonal with $\mathbf{K}(i,i) = 100k$, except for the element $\mathbf{K}(2,2) = k$.

From easy static balance considerations, we have $\mathbf{D}^T = (0\ -2a\ 0\ 0\ 0\ 0\ 0\ 0)$. Also,

$$\mathbf{G} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & -2a & 0 & 0 & -a & 2a & 0 & -3a & 2a \\ 2a & 0 & 0 & a & 0 & 0 & 3a & 0 & 0 \\ 0 & 0 & 0 & -2a & 0 & 0 & -2a & 0 & 0 \end{pmatrix};$$

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 2 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & -1 & -1 \end{pmatrix}.$$

The result of applying the above described algorithm for finding the basis of controllable internal forces to this example is $\mathbf{E} = (0\ 1\ 0\ 0\ -.5\ 0\ 0\ -.5\ 0)^T$, as it might be intuitively anticipated.

4 Design of the Grasp Cost Function

Equation (19) allows us to decouple the set of constraints on the choice of grasp forces discussed in section 2, and provides a means of describing all contact forces that balance a given external load while being realizable, in terms of a free variable vector, $\mathbf{y}$. This section will be concerned with the design of a cost function of such vector $\mathbf{y}$, so that the remaining constraints, namely minimum and maximum contact forces, and friction bounds, are satisfied at the cost function minimum.

We start by noting that all the constraints (6), (7), and (8) on the i-th contact force-torque, can be written in that order as

$$^j\sigma_i(\mathbf{y}) = {}^j\alpha_i\,||\mathbf{p}_i|| + {}^j\gamma_i\,\mathbf{f}_i^T\mathbf{n}_i + {}^j\delta_i \le 0, \qquad j = 1,2,3;, \tag{20}$$

where

$$\begin{array}{lllll} ^1\alpha_i = 0; & ^1\gamma_i = -1; & ^1\delta_i = f_{i,min}; & \text{min. force;} \\ ^2\alpha_i = 1; & ^2\gamma_i = 0; & ^2\delta_i = -f_{i,max}; & \text{max. force;} \\ ^3\alpha_i = \frac{1}{\sqrt{(1+\mu_i^2)}}; & ^3\gamma_i = -1; & ^3\delta_i = 0; & \text{slippage.} \end{array}$$

Let $^j\Omega_i \subset \Re^h$ indicate the open set of grasp variables that satisfy the constraint (20) of corresponding indices, $^j\Omega_i := \{\mathbf{y} \mid {}^j\sigma_i(\mathbf{y}) < 0\}$. For the i-th contact and the j-th constraint we consider the cost function

$$^jV_i(\mathbf{y}) = \frac{1}{2\,{}^j\sigma_i^2(\mathbf{y})} \tag{21}$$

and a global grasp cost function simply as the sum of local cost functions,

$$V(\mathbf{y}) = \sum_{i=1}^{n}\sum_{j=1}^{3} {}^jV_i(\mathbf{y}). \tag{22}$$

Such a cost function grows indefinitely as the constraint boundaries are approached, which is desirable in order to prevent their violation. Moreover, the V_i are minimized by contact forces that are further from the boundary, thus privileging choices of contact forces that correspond to most robust grasps. In order to show that such a cost function is well suitable as a performance index for the optimal control of grasp forces, its convexity with respect to the grasp variables $\mathbf{y}$ must be discussed.

The functions $\mathbf{p}_i = \mathbf{p}_i(\mathbf{y})$ and $q_i = q_i(\mathbf{y})$ can be written explicitly by partitioning the elements of (19) as

$$\mathbf{t} = \begin{pmatrix} \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_n \end{pmatrix} = \hat{\mathbf{t}} + \mathbf{E}\mathbf{y} = \begin{pmatrix} \mathbf{P}_1 \\ \vdots \\ \mathbf{P}_n \end{pmatrix}\mathbf{w} + \begin{pmatrix} \mathbf{M}_1 \\ \vdots \\ \mathbf{M}_n \end{pmatrix}\mathbf{y}, \tag{23}$$

so that we have

$$\mathbf{p}_i(\mathbf{y}) = \mathbf{P}_i\,\mathbf{w} + \mathbf{M}_i\,\mathbf{y}; \tag{24}$$

where $\mathbf{P}_i$ is a 3×6 matrix, and $\mathbf{M}_i$ is $3\times h$. The gradient of the cost function with respect to $\mathbf{y}$ can be written as the h-vector

$$\frac{\partial V}{\partial \mathbf{y}} = \sum_{i=1}^{n}\frac{\partial V_i}{\partial \mathbf{y}} = \sum_{i=1}^{n}\sum_{j=1}^{3} -\frac{1}{{}^j\sigma_i^3}\frac{\partial\,{}^j\sigma_i}{\partial \mathbf{y}}, \tag{25}$$

where the argument $\mathbf{y}$ has been dropped for brevity. Denoting with $\vec{\mathbf{v}} = \frac{\mathbf{v}}{||\mathbf{v}||}$ the *versor* of a generic vector $\mathbf{v}$, the gradient of $^j\sigma_i$ with respect to $\mathbf{y}$ is given by

$$\frac{\partial\,{}^j\sigma_i}{\partial \mathbf{y}} = {}^j\alpha_i\,\mathbf{M}_i^T\vec{\mathbf{p}}_i + {}^j\gamma_i\,\mathbf{M}_i^T\mathbf{n}_i. \tag{26}$$

The hessian of the cost function with respect to $\mathbf{y}$ is the $h\times h$ matrix

$$\frac{\partial^2 V}{\partial \mathbf{y}^2} = \sum_{i=1}^{n}\frac{\partial^2 V_i}{\partial \mathbf{y}^2} = \sum_{i=1}^{n}\sum_{j=1}^{3} -\frac{1}{{}^j\sigma_i^3}\frac{\partial^2\,{}^j\sigma_i}{\partial \mathbf{y}^2} + \frac{3}{{}^j\sigma_i^4}\frac{\partial\,{}^j\sigma_i}{\partial \mathbf{y}}\frac{\partial\,{}^j\sigma_i}{\partial \mathbf{y}}^T, \tag{27}$$

where

$$\frac{\partial^2\,{}^j\sigma_i}{\partial \mathbf{y}^2} = {}^j\alpha_i\,\frac{\mathbf{M}_i^T\left(\mathbf{I} - \vec{\mathbf{p}}_i\vec{\mathbf{p}}_i^T\right)\mathbf{M}_i}{||\mathbf{p}_i||}. \tag{28}$$

From (28) it can be easily observed that $\frac{\partial^2\,{}^j\sigma_i}{\partial \mathbf{y}^2}$ is positive semidefinite. Also, it can be shown that $^j\sigma_i(\mathbf{y})$ is a convex function. From the convexity of $^j\sigma_i(\mathbf{y})$ directly follows that $^j\Omega_i$ is a convex set. Assuming that at least one possible combination of grasp forces exists that complies with the given constraints, the set

$$\Omega = \bigcap_{i=1,\dots,n;}^{j=1,2,3;} {}^j\Omega_i$$

collecting all grasp variables $\mathbf{y}$ that satisfy every constraint, is not void and convex. Furthermore, from (27) it is easily verified that, for any $\mathbf{y}\in\Omega$, the cost function hessian is positive semidefinite. Hence, the cost function is convex in Ω.

This result is very useful, since it guarantees that it is possible to build grasp control laws that asymptotically converge to an optimal grasp (though not necessarily unique), provided that the starting grasp is acceptable ([Canon, Cullum and Polak, 1970]).

5 Grasp Optimization Algorithm

In the previous sections of this paper the grasp problem constraints have been analyzed, and linear equality constraints have been considered separately from nonlinear inequality constraints. This allowed the construction of a basis of all contact forces that comply with equality constraints, so as to eliminate those relationships from the actual computation of the optimal grasp forces. Inequality constraints have been dealt with by defining a cost function acting basically as a penalty function.

The aim of this section is to design a suitable law for controlling contact forces in the grasp of an object, which is subject to external disturbance forces $\mathbf{w}_d(t)$. Such disturbances are assumed unknown but bounded and resistible, i.e., there exists at least one possible solution to the grasp equation (2) with constraints (6), (7), and (8). The inputs to the integral manipulation system are assumed to be the actuator torques $\tau(t)$; moreover, the grasp is assumed static, i.e. the matrices $\mathbf{D}, \mathbf{G}$, and $\mathbf{K}$ do not vary as long as the grip on the object is held.

In both cases, we desire to obtain the goal while minimizing the cost function defined in the preceding section, so as to comply with the constraints on grasp forces, and maximize the grasp robustness. We assume in the following that both the desired interactions and the disturbance forces and torques are *feasible*, i.e. that there exists at least one possible solution to the grasp equation (2) with constraints (6), (7), and (8). The inputs to the integral manipulation system are assumed to be the actuator torques $\tau(t)$.

Claim. Assuming that a stable grasp is disturbed by forces and torques $\mathbf{w}_d(t)$ that remain resistible for any t, and have bounded derivatives $\|\dot{\mathbf{w}}_d\| \leq \bar{\omega}$, there exists a $\lambda > 0$ such that, for $\zeta > \lambda$ and for any positive definite $h \times h$ matrix $\mathbf{Z}$, the control law

$$\tau(t) = -\mathbf{D}^T \mathbf{G}^L \mathbf{w}_d(t) + \mathbf{D}^T \mathbf{E}\, \mathbf{y}(t); \qquad (29)$$

where

$$\mathbf{y}(t) = \int_0^t \dot{\mathbf{y}}(s)ds + \mathbf{y}_0; \qquad (30)$$

$$\dot{\mathbf{y}}(t) = -\zeta \mathbf{Z}\frac{\partial V}{\partial \mathbf{y}}, \qquad (31)$$

ensures that the object equilibrium is maintained, while asymptotically converging to an arbitrarily small neighborhood of an optimal (in the sense of (22)) set of grasp variables.

Proof. Consider the time derivative of the positive-definite Lyapunov candidate function V defined in (22),

$$\dot{V} = \frac{\partial V}{\partial \mathbf{y}}^T \dot{\mathbf{y}} + \frac{\partial V}{\partial \mathbf{w}}^T \dot{\mathbf{w}}_d = -\zeta \frac{\partial V}{\partial \mathbf{y}}^T \mathbf{Z} \frac{\partial V}{\partial \mathbf{y}} + \frac{\partial V}{\partial \mathbf{w}}^T \dot{\mathbf{w}}_d, \qquad (32)$$

where

$$\frac{\partial V}{\partial \mathbf{w}} = \sum_{i=1}^{n}\sum_{j=1}^{3} \frac{\partial^j V_i}{\partial \mathbf{w}},$$

$$\frac{\partial^j V_i}{\partial \mathbf{w}} = -\frac{1}{{}^j\sigma_i^3}\frac{\partial\, {}^j\sigma_i}{\partial \mathbf{w}},$$

$$\frac{\partial\, {}^j\sigma_i}{\partial \mathbf{w}} = {}^j\alpha_i\, \mathbf{P}_i^T \tilde{\mathbf{p}}_i + {}^j\gamma_i\, \mathbf{P}_i^T \mathbf{n}_i.$$

Let $C = \{\mathbf{y} \in \Re^h |\ \frac{\partial V}{\partial \mathbf{y}} = 0\}$ be the locus of optimising solutions. Note that $\dot{V}$ can be made negative outside an ϵ-neighborhood $B(\epsilon)$ of C for any bounded-derivative disturbance by choosing

$$\zeta > \lambda = \frac{\|\frac{\partial V}{\partial \mathbf{w}}\|\,\bar{\omega}}{\underline{z}\,\|\frac{\partial V}{\partial \mathbf{y}}\|_B^2}, \qquad (33)$$

where $\|\frac{\partial V}{\partial \mathbf{y}}\|_B = \min_{\mathbf{y} \notin B(\epsilon)} \|\frac{\partial V}{\partial \mathbf{y}}\|$, and $\underline{z}$ is the minimum singular value of $\mathbf{Z}$. Recalling that $\frac{\partial^2 V}{\partial \mathbf{y}^2}$ is positive semi-definite for any $\mathbf{y} \in \Omega$, the convergence to $B(\epsilon)$ is therefore guaranteed, q.e.d.

With regard to this control law, the following points should be noted:

- In (29), it is assumed that $\mathbf{w}_d(t)$ is known. This can be accomplished e.g. by using a wrist force/torque sensor, or simply summing the contact forces and torques measured by the contact sensors placed on the robot arm surfaces. Fig.2 shows a block diagram relative to this implementation.

  However, since force/torque sensing devices are rather slow, a detailed analysis of the effect of measurement time-lags on the control stability should be considered in this case. In alternative to measuring the disturbance forces, the optimal grasp control law (29) can be implemented using conventional joint position servos as depicted in fig.3.

  In fact, the position controllers provide torques that tend to restore the positional errors due to the disturbance $\mathbf{w}_d(t)$. Such torques cause a particular solution $\hat{\mathbf{t}}_d$ of the grasp balance equation (2) to be applied at the contacts. A suitable tuning of the stiffness matrix K (achieved through adjusting position servo gains) guarantees that the contact forces and torques due to the position controllers have no components in the null space of the grasp matrix $\mathbf{G}$. On the other hand, according to the discussion in section 3, optimizing grasp forces must be chosen in the null space of $\mathbf{G}$. Therefore, the optimizing torques can be superimposed to the position control torques, thus realizing an hybrid optimal grasp controller. Also note that a rather stiff joint position control will prevent significant perturbations of the grasp configuration, so that the static grasp assumption is not violated.

- In practice, starting with a "wrong" initial guess of $\mathbf{y}_0$, such that

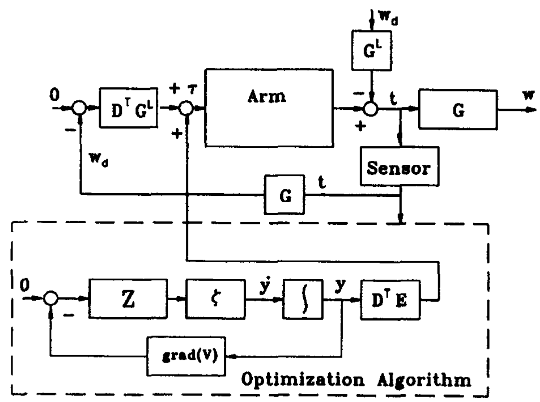

Figure 2: Block diagram of the optimal control method with direct disturbance compensation, based on force sensor feedback.

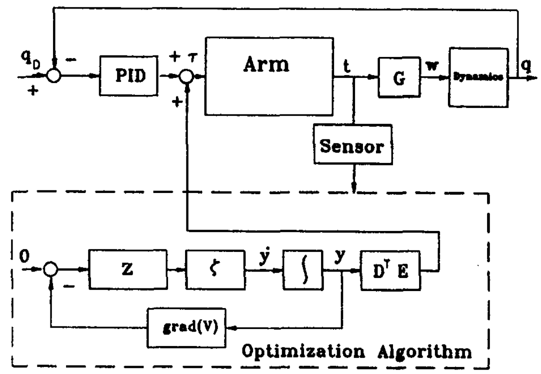

Figure 3: Block diagram of the hybrid optimal control method. The "dynamics" block refer to dynamical properties of the arm-object-environment system.

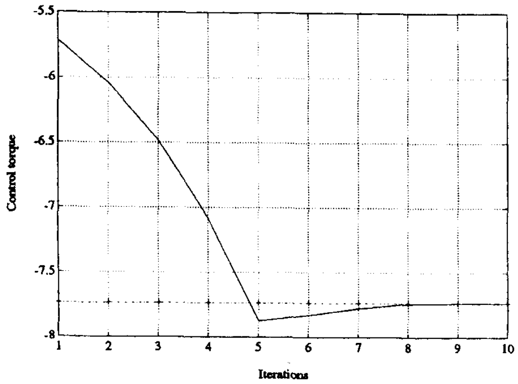

Figure 4: Plot of control torque history during algorithm iterations for the first example.

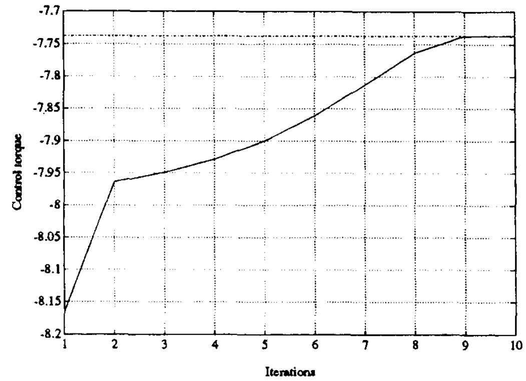

Figure 5: Plot of control torque history starting from an over-estimate of optimal contact forces.

some constraint is violated, may prevent the algorithm convergence and may cause the invalidation of the static grasp assumption. It is the task of a grasp planner to provide good initial guesses for grasp forces.

- Although the control algorithm has been discussed in the continuous time domain, it is straightforward to derive its discrete time analog. In this case, however, the global asymptotic convergence of the algorithm can be proven only for values of ζ smaller than a limit value, whose evaluation in real-time is not simple. A simple way to manage ζ is to reduce it whenever the cost function value at the current step is higher than the previous one. This simple method works well in almost all non-pathological cases, but there are cases where it does not converge. Such limitations on ζ will only allow the convergence to a finite neighborhood of the optimal grasp. However, the optimum is always reached at the steady-state equilibrium.
- The performance of the above optimization algorithm may be rather poor in high-dimensional problems. Substituting (31) with the solution of the system of linear equations

$$\frac{\partial^2 V}{\partial \mathbf{y}^2}\dot{\mathbf{y}}(t) = -\zeta\frac{\partial V}{\partial \mathbf{y}}, \tag{34}$$

provides a much faster convergent algorithm, although some care is needed in the vicinity of singularities of the hessian matrix.

6 Simulation

In this section simulation results relative to the simple example of fig.1 and to a more complex hand will be discussed, showing the dynamical performance of the proposed control algorithm.

Assume that the object grasped by the hand of fig.1 with $a = 1$ m, is subject to an external force step of intensity 4.0 N applied in the z direction at point $\mathbf{d} = (012)^T$ m. The applied disturbance results $\mathbf{w}_d = (4\ 0\ 0\ 0\ 8\ -4)^T$ N-Nm. Let the friction coefficients be $\mu_1 = \mu_2 = 0.7$, and the minimum and maximum contact forces be $f_{1,min} = f_{2,min} = 0.1$ N, and $f_{1,max} = f_{2,max} = 10.0$ N, respectively.

In fig.4 and fig.5 are shown the torque control input on the manipulator joint corresponding to the proposed control scheme output with different initial conditions. The optimal torque of -7.7371 Nm is reached at the steady-state after few iterations.

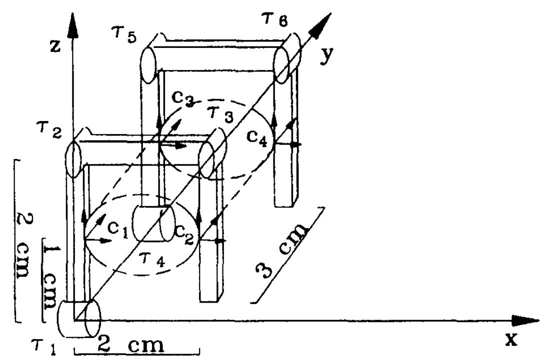

Figure 6: Power grasp with a two fingered hand.

As a second example, consider the hand depicted in fig.6, where two three-jointed fingers hold an object using their first and last links. Contact points are located at $\mathbf{c}_1 = (0\ 0\ 1)^T$ cm, $\mathbf{c}_2 = (2\ 0\ 1)^T$ cm, $\mathbf{c}_3 = (0\ 3\ 1)^T$ cm, and $\mathbf{c}_4 = (2\ 3\ 1)^T$ cm; the corresponding normal unit vectors are $\mathbf{n}_1 = (1\ 0\ 0)^T$, $\mathbf{n}_2 = (-1\ 0\ 0)^T$, $\mathbf{n}_3 = (1\ 0\ 0)^T$, $\mathbf{n}_4 = (-1\ 0\ 0)^T$. The $\mathbf{D}$ matrix relating contact forces to joint torques for this case is

$$\mathbf{D} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 \end{pmatrix},$$

while the grasp matrix is

$$
\mathbf{G}^T = \begin{pmatrix}
1 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & -1 & 0 & 2 \\
0 & 0 & 1 & 0 & -2 & 0 \\
1 & 0 & 0 & 0 & 1 & -3 \\
0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 3 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & -3 \\
0 & 1 & 0 & -1 & 0 & 2 \\
0 & 0 & 1 & 3 & -2 & 0
\end{pmatrix}.
$$

A basis of the null space of the grasp matrix is given by

$$
\mathbf{A} = \begin{pmatrix}
1 & 0 & 0 & -3 & 1.3 & 0 \\
0 & 1 & 0 & 2 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 5.7 & 0 & 0 \\
0 & 1 & 0 & -2 & -1 & -1 \\
0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 3 & -1.3 & 0 \\
0 & -1 & 0 & 2 & 1 & -1 \\
0 & 0 & -1 & 0 & 0 & 0 \\
-1 & 0 & 0 & -5.7 & 0 & 0 \\
0 & -1 & 0 & -2 & -1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}.
$$

Assuming for simplicity that the grasp stiffness matrix is diagonal with equal elements, $\mathbf{K} = k\mathbf{I}_{12}$, and applying the above discussed algorithm, we obtain a basis for the subspace of controllable internal forces as

$$
\mathbf{E} = \begin{pmatrix}
1 & 0 & 0 & -3 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 0 \\
-1 & 0 & 0 & 5.7 \\
0 & 1 & 0 & -2 \\
0 & 0 & -1 & 0 \\
1 & 0 & 0 & 3 \\
0 & -1 & 0 & 2 \\
0 & 0 & -1 & 0 \\
-1 & 0 & 0 & -5.7 \\
0 & -1 & 0 & -2 \\
0 & 0 & 1 & 0
\end{pmatrix}.
$$

In fig.7 a plot of the control torques generated by the law (29) with modification (34) is reported, simulating the response to an external force disturbance step of intensity 1 N applied in the z direction at point $\mathbf{d} = (1\ 1.5\ 1)^T$ cm. The friction coefficients are assumed equal everywhere to $\mu_t = 0.7$, and minimum and maximum contact forces are 0.1 N and 10.0 N, respectively, at each contact. Initial conditions on the optimizing vector $\mathbf{y}$ are $\mathbf{y}_0 = (200\ 0\ 0\ 20)^T$. The convergence of $\mathbf{y}$ to an optimal value of $\mathbf{y} = (573.3\ 146.1\ -40.8\ 204.5)^T$, corresponding to joint torques $\tau = (0\ 77.1\ 77.1\ 0\ 77.1\ )^T$ Ncm, is shown in the plot.

7 Conclusion

In this paper the problem of optimal control of grasp forces is analyzed, with particular attention to those manipulation systems that do not possess full mobility at each link involved in the grasp. For dealing with these systems, it is necessary to characterize the grasp forces that are internal and controllable: an algorithm for finding a basis of the subspace of such grasp forces is a contribution of this paper. An application of this result to the minimization of the slippage risk in a grasp is also presented, which exploits the convenient explicitation of linear constraints to obtain a control algorithm that efficiently converges to the optimal values of the joint torques. Finally, some simulations of the proposed algorithm have been presented.

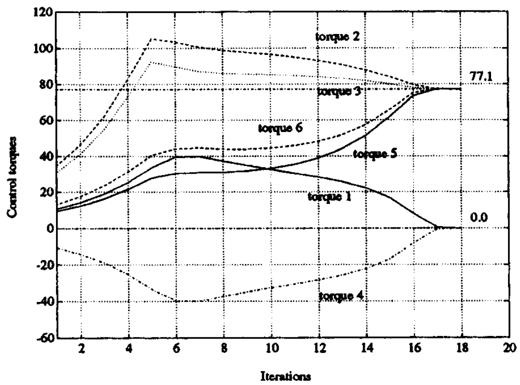

Figure 7: Plot of control torques for the second example.

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